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# An integrated method for the optimization of multiple-attribute product design

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# AN INTEGRATED METHOD FOR THE OPTIMIZATION OF MULTIPLE-ATTRIBUTE PRODUCT DESIGN

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A Dissertation

Presented to

the Graduate School of

Clemson University

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

Industrial Engineering

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by

Hsing-Pei Kao

May 1992

May 1, 1992

To the Graduate School:

This dissertation entitled "An Integrated Method for the Optimization of Multiple-Attribute Product Design" and written by Hsing-Pei Kao is presented to the Graduate School of Clemson University. I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy with a major in Industrial Engineering.

Dissertation Advisor

We have reviewed this dissertation and recommend its acceptance.

R. P. Davis

Accepted for the Graduate School:

## ABSTRACT

The interest of this research is to develop a method that facilitates product design. The method is confined to an analytical approach with a focus on consumer-oriented quality assurance. Two major subjects regarding product design are addressed by this study: (1) ascertaining the product's determinant quality attributes; and (2) ratifying the design that fulfills required quality.

For the development of this design method, several uncommon analytical tools are integrated into a systematic procedure. Since consumer judgement on quality is normally subjective, fuzzy sets methodology provides a reliable measurement for cognitive uncertainty inherent in human perception. A Monte Carlo simulation-based technique, namely JHE method, is used for processing fuzzy numbers. Quality attributes from consumer perspective are ranked by an entropy method, while ranking from designer perspective is performed by an eigenvector method. These two attribute priorities are then integrated to designate the determinant quality attributes. Since a product's quality is dependent upon its design specification, multivariate regression analysis is applicable for modeling the correlation between determinant quality attributes and design factors. Whereas linguistic scales are used for measuring quality, entropy method and multivariate regression analysis are incorporated with the JHE method.

The resulting fuzzy regression model is used to experiment with various design parameter settings, so that consumer preferential utility, which reflects quality level of the designed product, can be predicted. In the comparison of the design alternatives, the optimal design can be consequently ensured. According to the procedure, computer programs are developed for processing data and validating the design method.

# DEDICATION

To my wife, Hui-Ling.

No exact words can show my love and gratitude to my wife Pang Hui-Ling, who gives me all her best, especially our precious daughter Justine Shao-Tang. Without Hui-Ling's love and patience, I would not have been able to complete this work.

A Chinese proverb says "the highest filial piety is to have parents' good names hold in esteem." My special gratitude go to my parents Professor Kao Shu-Fei and Mrs. Kao Lee Wu-Ying, parents-in-law Mr. Pang Jin-Yu and Mrs. Pang Chao Re-Hua, and other family members for their love and support. I also want to express my deepest memory to my grandfather Mr. Kao Hsiao-Soun.

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# CHAPTER I

# INTRODUCTION

## Background

In this era of intensifying global competition, industries are exploring diverse ways to acquire a competitive advantage to protect or improve their market positions. It is believed that producing products superior in quality is a more promising strategy than others, such as competition by price reduction that may drive a company's profits down to a detrimental level. This strategy of improving quality is supported by the finding of an American Society for Quality Control (ASQC)/Gallup survey that a product's quality is the most important factor in consumer's purchase decision (18).

In a traditional scheme for quality improvement, the product quality is usually defined by the manufacturer. This scheme is too narrow in scope because thinking of quality only as a function of exacting manufacturing standards is a denial of the consumer's existence and desires. Industries have often been criticized that the product development is usually accomplished without explicit consumer input, and it easily results in market failures (55). Therefore, a more proactive strategy should be adopted to utilize consumer inputs throughout the process of product development.

Consumers use a product and react to its ability to satisfy their needs. They have little knowledge of, or concern for, the technical standards in its manufacture. The manufacturer should realize that consumer preferences for physical aspects of the product may or may not be related to currently established technical measures of product quality. In order to succeed in the market, the manufacturer must know what consumers want and how to design a product to meet their preferences.

Consumer involvement should have been a major part of quality improvement plans; as W. Edwards Deming stated, "the consumer is the most important part of the production line" (11). This idea is not really new in design theory. In fact, the concept of consumer-defined quality can be traced back to Feigenbaum when he defined quality as "best for certain customer conditions" (16). Also in the area of marketing research, consumer behavior has been studied rigorously and product quality has never been a trivial issue. The problem is how to substantiate this idea by letting designer and engineer hear and understand the voice of customer, and thus respond to it effectively.

The emphasis on consumer involvement as a renaissance in the quality area can be seen in 1984's report to the American Society for Quality Control (ASQC) (25):

- It is not those who offer the product but those whom it serves that have the final word on how well a product fulfills needs and expectations.
- Satisfaction is related to competitive offerings.
- Satisfaction is formed over the product lifetime.
- A composite of attributes is needed to provide the most satisfaction to those whom the product serves.

Quality to the consumer should be the most significant measure of quality improvement. Furthermore, quality is defined comparatively - relative to competitors - rather than against fixed, internal standards. Acquiring the right specification for design parameters is actually the leading requisite for product development, while conforming to specifications in manufacture is the result. Otherwise, excellence only in manufacturing process is futile when it conforms to a faulty specification (19). Whether the design is right or faulty is actually judged by the consumer; therefore, consumer input is most valuable for product design.

From the consumer standpoint, quality is "perceived" as the relationship between expectations and a product's ability to meet these expectations. Consumer satisfaction is achieved when expectations are met or favorably exceeded. Nonetheless, because products and consumers become more and more sophisticated, it is a difficult task to assess accurately, not to mention to respond effectively, to the expectations or requirements from the consumer. In spite of this difficulty, product innovation with a focus on consumer-oriented quality assurance should be the major concern in modern manufacture.

Product quality perceived by the consumer can be categorized into three areas: product requirement specification and design quality, conformance to design, and product performance (24). It is apparent that quality must be designed into the product and it is not a feature that can be added by a process, such as inspection. Industries need to moved their focus from manufacturing process quality control to product development quality assurance. This prevailing concept has been presented as "off-line" or "upstream" quality control in contrast to traditional "on-line" quality control (17). It should be noted that this new approach does not imply that statistical process control (SPC) is no longer important; it simply directs the significance of extending quality concern to an earlier stage of production, i.e., product design.

Design assurance is defined as "those planned and systematic actions taken to provide confidence that the product design will satisfy the requirements of its intended use" (7). Design is driven by the function, the object, or situation it is intended to fulfill. Quality either defined as "fitness for use (30)" or "conformance to requirements (10)" is determined mainly in the design phase.

Design takes place within a triangle of constraints: the end use of a product, the materials of which it is made, and the tools and processes by which it is made (42). This triangle forces the market researcher, the designer, and the engineer to collaborate. While the market researcher tends to envision a product as a general concept, the designer concentrates on technical details. To synthesize designing with marketing, effort should be directed towards their interface. It becomes a new challenge to the engineer to bridge the breach between marketing and product design.

To have a product prevail in a targeted market requires more than the functional aspects of the product. Normally, every product has multiple characteristics that both the consumer and the manufacturer are concerned with. The design of a product must address its usability, ergonomic tractability, technical and economic viability, aesthetic sensibility, and image congruity (42). In a similar framework, eight dimensions of quality have been proposed to serve for strategic analysis, which include performance, features, reliability, conformance, durability, serviceability, aesthetics, and perceived quality (20).

However, it is not practical to the manufacturer to pursue all the dimensions simultaneously unless it intends to charge unacceptably high prices (21). Furthermore, technological capability and manufacturing cost may impose additional constraints. Therefore, the manufacturer should pursue selective quality dimensions or attributes to concentrate one's design effort. Product design usually entails some form of compromise among conflicting requirements arising from various dimensions, and the focused dimensions should be important to the consumer. The essence of this strategy is consistent with the Pareto principle that the "vital few" factors have the greatest reward.

An essential factor for any effective design assurance is that consumer needs and expectations be understood. Every aspect of the manufacturer's endeavor should be leveraged by the consumer preference of product's features and quality characteristics. Determining the level of acceptability for each quality characteristic is the principle of practical and effective design. In summary, there are three primary issues that require solutions should be addressed:

- measure of consumer preference,
- identification of quality characteristics that are important to both the consumer and the manufacturer, and
- utilization of consumer input for the design specification.

To act upon these issues, we need a design methodology that provides a structured process to solve problems, and this methodology should be able to minimize the risks associated with decisions made under uncertainties.

## Statement of the Problem

For the manufacturer, establishing design specifications is most challenging in product innovation. Most often the problems confronted in developing new products are (1) defining the right product at the right price, (2) getting a firm up-front definition of what the product is, and (3) getting specifications that meet the requirements (45). In order to solve these problems, the issue of relating design specification and consumer perception needs to be addressed.

In general, every product possesses more than one characteristic or attribute, and product attributes are normally shared by other brands. Multiple attributes imply that a product can be differentiated in a multitude of ways. For example, the quality of a personal computer is a multifaceted variable that may be distinguished by its safety, data integrity, system integrity, reliability, compatibility, functionality, performance, and cost. Different PC buyers, even computer experts, may have different priorities for these attributes (54).

The product, per se, does not provide utility to the consumer; attributes of the product give rise to utility. In consumer theory, economists use a utility function to characterize consumer preference: $U(\mathbf{Z}) = f(z_1, z_2, \ldots, z_n)$, where $z_j$ is an attribute's scalar that satisfies an individual, and $U(\mathbf{Z})$ is the utility associated with the product that the consumer wishes to maximize (41). The concept of this model, shared by various marketing studies, is that consumer preference should be modelled in a multi-attribute framework. A primary goal of these studies is to identify the association between attributes, or the attribute ranks (55).

From the consumer standpoint, product attributes should be discernible and can be judged in grades. It is noted that different attributes may not contribute the same level of utility to a consumer; and a specific attribute may not bring the same utility level to different consumers. Take an automobile as an example; different consumers may have different degrees of concern on acceleration, economy, ride, handling, capacity, and appearance. Only by aggregating individual consumer's opinions on the multiple attributes inherent in a product can the overall products' quality be objectively measured and the consumer preference be ascertained.

One apparent source of information seen by the manufacturer is consumer complaints on the product's quality. Consumer complaints in fact do minimal good for quality improvement because the cost of remedying a problem increases exponentially as the work has progressed into the development life cycle (43). A valuable consumer input that can be utilized is the consumer's evaluation of in-market products. From the consumer evaluation profile, the product's determinant attributes can be identified with more ease.

Since utility is sensed or perceived when a product is used by a consumer, that utility reveals the value or merit of a combination of quality attributes. As mentioned before, a product can be evaluated if its attributes are distinguishable. It is noted that consumers tend to evaluate products based on individual subjectivity. The words or phrases that the consumer uses to grade a product based on certain attributes are usually in the form of qualitative scales, such as "The ride of this car is excellent, but its fuel economy is fairly poor." This human subjectivity causes the difficulty of objective measurement. A reliable measurement method has to be employed for quantifying "cognitive uncertainty" inherent in human judgement in order to insure a credible analysis.

The presence of a multiplicity of attributes makes the analysis even more complex. A company that determines to compete based on quality needs to improve the determinant product attributes, which are governed by the aggregated consumer preference priority and subject to the company's technical and cost limitations. Manufacturers are forced to trade off some attributes against others, corresponding to what the targeted market has determined.

It should be recognized that consumers respond to, and benefit from, the product's services, rather than initiate the idea of the product's design. In general, technical specification of design is not of interest to consumers. Even the consumer requirements are clearly known, the need for response forces the manufacturer to transform consumer preferences into counterpart design requirements. This design task is accomplished by identifying critical product and process parameters and specifying an optimal set of measurable design parameters.

In summary, the leading task for product development is to design a product that is right for the targeted market. In the design phase, the most essential step is knowing what product attributes are important and where tradeoffs can be made to save available resources. Once consumer preferences are recognized, they need to be translated into design or improvement actions. The design effort ends when the product's specifications are "cast in concrete."

A generally accepted strategic framework for product development consists of three major sequential steps: (1) determine the targeted market and methods of collecting consumer information; (2) identify the determinant product attributes; and (3) respond to market demands with optimal design (61). In agreement with this strategic framework, the objective of this research is to develop a quantitative method to assist in (1) defining quality from the consumer standpoint, (2) modeling the correlation between the defined quality and design specification, and (3) identifying the most desirable design alternative.

Significance of the Research

There are several significant aspects of this study. First and foremost, this research provides an analytical method that integrates marketing and product design in a systematic procedure, which generates a working model that can be utilized by the designer. Similar subjects have been addressed by numerous studies, but none of them is capable of relating consumer perception on quality directly with product design from the engineering aspect, and most of them only provide abstract managerial guidelines.

The second significant aspect of this study is the use of fuzzy sets methodology to provide a more reliable measurement for consumer perceived quality. The traditionally used measurement techniques in marketing are basically simple mapping between linguistic scales and numbers, whcih ignores the cognitive uncertainty inherent in human perception (27). The application of fuzzy sets has been found extremely powerful in the areas that involve human judgement and decision making. These areas of study include the most advanced artificial intelligence technology, medical diagnostics, engineering, psychology, and social sciences. It is foreseen that fuzzy sets methodology will soon be emphasized in marketing research.

Ranking attributes analytically is preferred to arbitrarily selecting attributes purely based on the designer's experience. The third significant aspect of this study is that it provides ranking procedure on products as well as on product attributes by taking the judgements from both the designer and the consumer into consideration. This procedure basically applies multiple attribute decision making (MADM) theory as a framework that operates on qualitative data. It is designed to manage the individual preference difference and the results of it are objective measurement of quality and ranking of products and attributes.

The fourth significant aspect of this research is that a mathematical model will be generated from the method. This model describes the correlation between consumer perceived product attributes and design parameters; it is a working model that assists translating consumer requirements into design specifications. By implementing experimentation on this model, the optimal combination of design parameters can be identified, and accordingly an ideal product design is specified.

## Structure of the Research

The results of this research are presented in five additional chapters. Chapter II presents the literature review, which reviews the previous methods that address the subjects of product design, quality assurance, and consumer preference analysis. Chapter III contains specific definition of the research subject and proposes methods for solving problems. Chapter IV presents the methodological structure and describes detailed procedure for analysis. According to the procedure, several computer programs are coded. An illustrative example will also be given to demonstrate the design method. In Chapter V, the issues of validating the method will be addressed. In Chapter VI, conclusions and recommendations for further research will be presented.

# CHAPTER II

# LITERATURE REVIEW

In addressing the subject of product quality, both engineering and marketing researchers have made remarkable efforts in developing theories and methods. For the concentration of this research, the following review outlines the most prominent methods in quality engineering, consumer study, and product development.

Whereas Taguchi methods comprise a set of engineering-oriented analytical tools, conjoint analysis and multidimensional scaling are marketing-specific techniques. Quality Function Deployment is a company-wide quality control system which deploys responsibilities to all departments in a manner of collaboration. The fundamental characteristics and principles of these methods will be discussed respectively. In the last section, a general commentary will be presented.

## Quality Engineering: Taguchi Method

The fundamental concepts of Taguchi methods are: (1) product's quality can be quantified in terms of the total loss to the society; and (2) quality must be designed into the product to achieve high quality levels economically. To Taguchi, quality is measured via the "loss function" in which the financial loss, associated with product's quality, and functional specification are united through a quadratic relationship: $L(y) = k\,(y - t)^2$, where L is the loss when the functional (quality) characteristic is equal to y as the nominal (target) value is $t$, and $k$ is a constant (59). Quality loss is reduced through the continuous reduction of variation, even within the allowed tolerance limits. Quality is best when product characteristics are at target values; hence, the loss to society is minimal through the product's life cycle.

According to Taguchi, a product's performance is affected by so called noise factors, which are are difficult, impossible, or expensive to control. There are three types of noise factors: external (i.e., temperature, humidity, etc.), manufacturing imperfection (i.e., cause of part-to-part variation), and deterioration (58). One economic approach to reducing a product's functional variation is to center the design parameters in such a way to minimize sensitivity to all noise factors. The design parameters are the factors over which the designer has direct control and whose level or value is specified by the designer.

Taguchi advocates a three-stage design process: system design, parameter design, and tolerance design (31, 44, 46). System design is the process of applying scientific and engineering knowledge to produce a basic functional prototype design, which defines the initial settings of product design factors. Parameter design is considered the most crucial step in developing stable and reliable products. It specifies the levels of control factors that minimize sensitivity to all noise factors. In other words, parameter design is an investigation conducted to identify settings that minimize the performance variation. If parameter design fails to produce adequately low functional variation of the product, then, during tolerance design, tolerances are selectively reduced on the basis of cost effectiveness. At this stage, quality is improved by tightening tolerances on product or process parameters to reduce the performance variation.

For optimizing product design and manufacturing processes, Taguchi methods depend heavily on statistical concepts and tools, especially design of experiments. In the design of experiments, Taguchi uses orthogonal arrays for identifying settings of design parameters that maximize a performance statistic. This procedure is summarized as follows:

1. Identify the initial and competing settings of factors and their ranges, as well as the noise factors that cause the performance variation.

2. Construct the design and noise matrices by using orthogonal arrays, and plan the parameter design experiment.

3. Construct the parameter design experiment and evaluate the performance statistic, such as signal-to-noise ratio, for each test run of the design matrix.

4. Use the values of the performance statistics to predict new settings of the design parameters.

5. Confirm that the new settings indeed improve the performance statistic by follow-up experiments (31).

To summarize, reducing sensitivity to variation is a main force of Taguchi methods. Sensitivity to variation is reduced by adjusting factors that can be controlled in a way that minimizes the effects of factors that can not be controlled. This results in what Taguchi calls a "robust" design in which the loss due to variation of performance from the target is reduced.

## Conjoint Analysis

The usual problem of optimizing product design is that preference for various attributes may be in conflict or there may not be enough resources to satisfy all the preferences. It usually requires a compromise set of attribute levels. Conjoint analysis is a survey-based technique for measuring consumers' trade-offs among product attributes, and it is the most extensively used analytical tool in marketing research since it was introduced (22, 23).

Conjoint analysis provides a quantitative measure of the relative importance of quality attributes (1). It is applied to understand how consumers make choices within an existing market, coupled with information on the perceptions of the competitive products; and to study the linkage of product feature to consumer perception, which suggests product configurations with significant consumer preference (61). The conditions of applying conjoint analysis are:

. the alternative products have a number of attributes and each with two or more levels,

. most of the feasible combinations of attribute levels do not presently exist, and

. the range of possible attribute levels can be expanded beyond those presently available (1).

The conjoint model involves the assumption that preference can be modeled by adding the utilities associated with attribute levels. Data is collected by giving respondents profiles of product offerings, and each profile is made up of a set of attribute levels. Respondents are asked to make trade-off judgements on attribute pairs or to make an overall judgement of a full profile of attributes. Each level of individual attribute is assigned utility or worth in numerical value by respondents.

There are two major findings from the analysis. First, the rank order of the respondents' preference of product profiles is identified, according to the comparisons of profiles' totalled utilities. Second, the importance of attributes are found based on the rule that the greater the difference between the highest and lowest valued levels of an attribute, the more important the attribute. Conversely, if all the possible levels have the same utility, the attribute is not important, for it has no influence on the overall attitude.

## Multidimensional Scaling

Much of marketing management is concerned with the question of positioning products in the targeted market. Multidimensional scaling (MDS) addresses the general problems of positioning products in a perceptual space: identify competitors, compare with competitors based on certain attributes, and develop positioning strategy.

MDS is concerned primarily with the spatial representation of relationships among behavioral data such as consumer perceptions and preferences. It involves identifying product attributes by which products are perceived and evaluated, and positioning those products and ideal products with respect to those attributes. In other words, the MDS analysis provides a perceptual map of the products on the attributes (9). For data collection, a group of respondents are asked to rate each of the products, using a set of pre-specified scales, on the identified attributes. An average rating of the respondent group on each attribute would be of interest.

It would be more useful if the attributes could be combined into two or three dimensions or factors. There are two major MDS techniques that are used to reduce the attributes to a small number of factors: factor analysis and discriminant analysis (61). Both techniques are actually multivariate statistical methods.

Factor analysis is used to investigate the structure of the evaluation space common to a group of decision participants, and to reduce the perceptual variables (attributes) linearly into a small set of independent composite factors. The factor analysis model considers that the variance observed in each original variable is partly accounted for by a set of common factors, and partly by a factor specific to that variable. The common factors account for the correlations observed among the original variables. If in a specific application, a few of the factors extracted account for the major part of the total variance, these independent composite factors actually reproduce the patterns of intercorrelations among original variables.

Whereas the goal of factor analysis is to generate composite factors, the goal of discriminant analysis is to generate dimensions that will discriminate or separate the products as much as possible. Discriminant analysis identifies clusters of attributes on which products differ. As in factor analysis, each dimension is based on a combination of the underlying perceptual variables or attributes in discriminant analysis. However, in discriminant analysis, the extent to which an attribute will tend to be an important contributor toward a dimension depends on the extent to which there is a perceived difference among the products on that attribute. The second characteristic of discriminant analysis is that it provides a test of statistical significance. The test will determine the probability that the between-object distance was due simply to a statistical accident.

## Quality Function Deployment

Quality Function Deployment (QFD) means that responsibilities for producing a quality product must be assigned to all parts of a company (32). It is a system for translating consumer requirements into appropriate technical requirements at each stage from research and product development to engineering and manufacturing. Quality control carried out in this manner might be called a company-wide quality control or total quality control.

QFD begins with market research that identifies things that consumers prefer, which is called the voice of the customer (VOC). Through the QFD process the VOC is boiled down into part specifications and manufacturing parameters an engineer can act upon. This way, QFD ensures engineering activities are focused on meeting the VOC.

For the product development, a design approach is needed to find means to achieve specific product objectives. This design approach requires companies to work back from objectives to means of achieving those objectives. It gradually breaks down objectives into segments and connects each segment with particular means of accomplishing that segment. It is therefore called a system of moving from upstream to downstream.

QFD 's activities can be grouped into two fields: "product quality deployment" and "deployment of quality function". In product quality deployment, the consumer requirements are converted into counterpart characteristics, thus determining the design quality of the final product, and further, systematically deploying them to the quality of each part as well as functional component, in relation to process element. The deployment of quality function refers to the activities needed to assure that customer-required quality is achieved. It is to deploy quality-related job functions step by step with both the series of objectives and means down to the finest detail. Even after counterpart quality characteristics have been set, each department must be assigned specific responsibilities.

There are four key documents that the QFD system is based upon (56):

1. Overall Customer Requirement Planning Matrix: provides a way of turning general customer requirements, drawn from market evaluations, into specified final product control characteristics.

2. Final Product Characteristic Deployment Matrix: translates the output of the final product control characteristics into critical component characteristics.

3. Process Plan and Quality Control Charts: identify critical product and process parameters, as well as control or check points for each of those parameters.

4. Operating Instructions: identify operations to be performed by plant personnel to assure that important parameters are achieved.

QFD system and Taguchi methods could be complementary to each other (15, 47). While QFD can help identify key product or process concerns with respect to customer requirements, Taguchi methods can help identify what product or process relationships truly exist as well as the nature of the relationship.

## Critique

Although marketing research may provide valuable information to the designer about consumer requirements, they do not direct the designer on how to respond to the requirements by specifying design parameters. Despite the strength of QFD system and Taguchi methods in quality improvement, they have several weaknesses as discussed below.

The first weakness of marketing research is that although important quality attributes can be identified, the relationship between attributes and design parameters has not been addressed. Therefore, it leaves the gap between marketing and designing unfilled.

The second weakness of marketing research concerns the specified scale used in surveys. It is a common practice in consumer study to map a consumer's evaluation, e.g., "good", with a number, such as 5 of a 7-point scale, for computational convenience. This mapping may not be reliable because it tries to capture human perception by using discontinuous scales. In order to achieve better results, an improved method is needed to provide a continuous scale for capturing consumer preferences.

In addition, in most marketing studies, the terms "attribute", "response variable", "factor", and "dimension" are used interchangeably that causes much confusion. It is more appropriate to use "attributes" and "response variables" to refer to quality characteristics that consumers can perceive and evaluate, and "factors" and "dimensions" refer to the parameters that can be controlled by the designer.

Most Taguchi applications to date have been concerned with the optimization of a single-response product or process, and its application to multiple-response cases may be very complex (37). The neglect of the coexistence of product's multiple characteristics is actually more crucial than the neglect of the interaction between design parameters that has been often criticized. For instance, fuel economy and comfort are two major attributes of a car that may conflict with each other. A comfortable car usually has a larger build and is heavier that normally lead to poorer fuel economy. It would be unwise to make a car extremely economic in fuel consumption when comfort would be sacrificed. Taguchi methods may be justified for application when the independence between product's attributes is certain or the selected attribute is the only concern.

The second weakness of Taguchi methods in product optimization is that the quality characteristic is selected without obvious consumer input. It is nice to reduce the functional variation of a product's attribute, but it could be a waste to optimize an attribute that is of no significance to the consumers.

The third weakness of Taguchi methods is about the measurement of selected attribute. Whereas the nominal value of quality characteristic is specified numerically in Taguchi's loss function, consumers express their concerns about product's characteristics in subjective terms. However, the issue of converting the subjective consumer voice into corresponding measurement has not been addressed by Taguchi methods. In such a manner, consumer requirements do not have direct and apparent influence on the specification of nominal parameter value.

The QFD system has the strength to force all the departments of a company to be concerned about quality. Nevertheless, it is an indirect method that translates the consumer voice to the design parameters. Any mistranslation or misinterpretation at the transient stages could bias the information that a designer receives. Furthermore, the use of symbols for indicating the degree of relationship between consumer requirements and control characteristics in a relationship matrix could be even less reliable than the number mapping method.

Although the reviewed methods have their weaknesses, they are powerful tools for specific studies. This research benefits from many insightful ideas presented in these methods. For instance, utility concept and product's multiple-attribute nature exhibited in marketing research reveal the notion of consumer preference to the products, and Taguchi's loss function illustrates the existence of various quality levels. Furthermore, Taguchi's experimental design guides this research in modeling the relationship between product attributes and design factors. As a managerial tool that focuses on the translation of consumer requirements into design actions in a logical procedure, QFD provides the philosophical framework to this research.

# CHAPTER III

# RESEARCH METHODOLOGY

The objective of this research is to develop a quantitative method that extracts information from consumer inputs to facilitate product design specification. In this chapter, the research problem will be defined specifically, and the solution approach as well as the fundamental theories of the application methods will be presented.

## Problem Definition

When a product is shipped to the market, the consumer becomes the final judge of its quality, whereas quality level is already determined by how it was designed and manufactured. It will be advantageous if an explicit model is available in describing the relationship between consumer perceived quality and design specification at the design phase. It is the primary objective of this research to provide a method that will generate a relationship model between a product's quality attributes and design factors. In this research, quality attributes are defined as the product characteristics discernible to the consumer and design factors are defined as the physical dimensions that the designer can control and specify.

Using the model generated from the method, the designer can experiment with different design parameter settings to predict the corresponding consumer preferential utility. Consequently, an optimal design can be identified for forming the product's prototype. In a sense, this relationship model is a conceptual blueprint to be used to direct product design in order to meet consumer preference.

Based on the notion that a product's quality can be "perceived" and "verbalized" by consumers while quality is affected by design, the relationship of design specification, product quality, and consumer perception can be described as a "cause-effect-response" chain (see Figure 3.1).

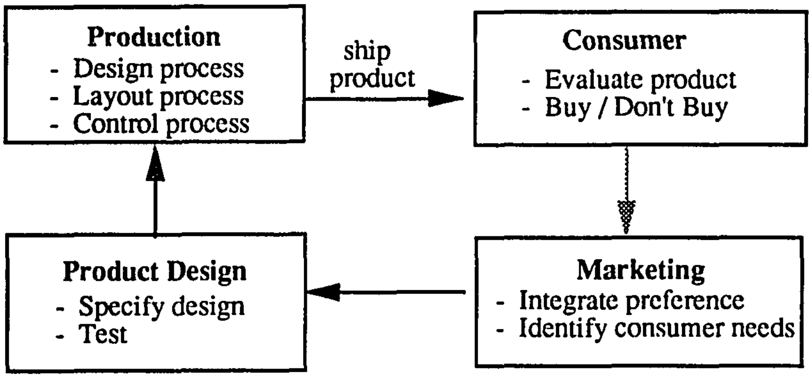


Figure 3.1 Chain of cause-effect-response as design-quality-perception relationship.

In addition, there are two assumptions that are applicable to most consumer products: every product has multiple quality attributes, according to which the product is evaluated, and a number of product alternatives compete in a targeted market. In the following, three interrelated subjects are addressed for specifying the problem: (1) measurement of consumer perception, (2) identification of determinant quality attributes, and (3) product design specification.

## Measurement of Consumer Perception

Consumer opinion is normally expressed in subjective and natural terms which are designated as "verbal hedonic scales" by marketing research in the "consumer acceptance tests" (38). In a five-point scheme these scales may include terms such as "excellent", "good", "fair", "poor", and "terrible". As long as the verbal scales are sensibly discriminative in terms of grade or intensity, the acceptance test can be designed as seven-point or nine-point scheme. These scales are considered as the linguistic measures of the preferential utility that consumer perceives a product or its attributes.

For the purpose of modeling the relationship between quality attributes and design factors in a quantitative manner, the measurement of these linguistic scales is required, where measurement is defined as "the procedure for assigning the real numbers to objects to represent quantities of objects' attributes (1)." The problem of measuring or quantifying subjective perception has been addressed in diverse fields, such as psychology, marketing, and human factor engineering, where human judgment is involved; and various methods have been proposed. In general, the underlying assumption of these methods is that "meanings" of words can be specified as points along a numerical scale and the variability about the scale value is ascribable to the statistical nature of the system (27).

Nonetheless, researches in semantic theory have brought into question the assumption that meanings can be precisely represented by numbers. Instead, it has been proposed that natural language terms are to a lesser or greater degree inherently vague, such that, the boundary of a term is never a point but a region where the term gradually moves from being applicable to nonapplicable (34, 35). Furthermore, between two adjacent subjective terms, such as "good" and "excellent" there may exist a certain degree of overlap in meaning that is not indicated by simple numerical mapping.

## Identification of Determinant Quality Attributes

Determinant quality attributes are those which both distinguish the product alternatives in the competitive market and can be reliably associated with consumer preference (2). When a product is evaluated in terms of multiple attributes by consumers, the following phenomena may be found: (1) different consumers may not acquire the same magnitude of utility from an quality attribute; (2) individual consumer may trade off some attributes to get others; (3) evaluated attributes may be independent, interdependent, or conflicting with one another. To the manufacturer, it is not realistic or possible to produce a product that meets every consumer's preference priority. Thus, a ranking method is needed to specify the determinant quality attributes that are significant to the targeted market.

Although the quality attributes' degrees of importance should be determined by consumers, the designer's opinion is also valuable due to his knowledge and experience. To the designer, the concept of a product is the composition of a set of quality attributes that are interested by product users. Therefore, the set of attributes that the designer evaluates should be the same set of attributes evaluated by consumers. Suppose the priorities from consumers and the designer are different, a compromise between them is needed.

## Product Design Specification

The designer's task is to specify the physical design. The design is usually performed according to the presumptive association between consumer preference and design specification. The traditional approach may go through the process of trial and error by building and experimenting costly prototypes to accomplish a design that may not be preferable to the consumer. It would be beneficial if a descriptive model is available in describing the perception-design correlativity so that experimentation can be implemented on the model for locating the optimal design.

## Approach to the Problem

According to the defined problem, the solution procedure includes attribute analysis and model development. Before addressing the solution methods, it is necessary to investigate what input data is needed for analyses.

When a consumer's buying decision is made by evaluating a number of $P$ product alternatives ($A_k$, k = 1,2,...,$P$) according to $M$ quality attributes ($QA_i$, i = 1,2,...,$M$), his or her evaluation can be presented as a decision matrix **C** as shown below. If there are a number of $t$ consumers evaluate these product alternatives, a number of $t$ decision matrices can be constructed.

$$
\mathbf{C} = \begin{array}{c} \\ QA_1 \\ QA_2 \\ \bullet \\ \bullet \\ \bullet \\ QA_M \end{array}
\begin{array}{c} \begin{array}{cccccc} A_1 & A_2 & \bullet & \bullet & \bullet & A_P \end{array} \\
\begin{bmatrix} \text{fair} & \text{poor} & \bullet & \bullet & \bullet & \text{good} \\ \text{poor} & \text{excel} & \bullet & \bullet & \bullet & \text{fair} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \text{good} & \text{fair} & \bullet & \bullet & \bullet & \text{bad} \end{bmatrix} \end{array} .
$$

The designer is assumed to be a single decision maker who should be neutral to in-market products. Thus, from the designer's viewpoint, quality attribute ranking procedure should be independent from product alternatives. Among quality attributes, the designer compares every attribute with another to evaluate their relative importance. From this procedure of pair-wise comparison a decision matrix **D** can be built as below.

$$\mathbf{D} = \begin{array}{c} \\ QA_1 \\ QA_2 \\ \bullet \\ \bullet \\ \bullet \\ QA_M \end{array} \begin{array}{c} \begin{array}{cccccc} QA_1 & QA_2 & \bullet & \bullet & \bullet & QA_M \end{array} \\ \left[ \begin{array}{cccccc} EI & DI & \bullet & \bullet & \bullet & AI \\ & EI & \bullet & \bullet & \bullet & SI \\ & & \bullet & \bullet & \bullet & WI \\ & & & \bullet & \bullet & \bullet \\ & & & & \bullet & \bullet \\ & & & & & EI \end{array} \right] \end{array} .$$

Each element in the matrix is a linguistic term of the relative importance, e.g., equal importance (EI), weak importance (WI), strong importance (SI), and so on.

It is assumed that design factors are measurable and the specifications of in-market product alternatives are obtainable. A data file, in which there are *P* product alternatives with *N* common design factors, can be formed as below.

$$\begin{array}{c} \\ DF_1 \\ DF_2 \\ \bullet \\ \bullet \\ \bullet \\ DF_N \end{array} \begin{array}{c} \begin{array}{cccccc} A_1 & A_2 & \bullet & \bullet & \bullet & A_P \end{array} \\ \left[ \begin{array}{cccccc} 12 & 11 & \bullet & \bullet & \bullet & 13 \\ 6 & 9 & \bullet & \bullet & \bullet & 7 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ M & N & \bullet & \bullet & \bullet & G \end{array} \right] \end{array}$$

Once input data on quality attributes and design factors is obtained, the subsequent task is to perform analyses for ranking quality attributes and developing relationship model. In the following discussion, the application methods for analysis will be proposed.

## Attribute Analysis: Multiple Attribute Decision Making

Decision making is the process of selecting an optimal course of action from the available alternatives, and the process is usually formulated in a mathematical model. Ranking quality attributes is considered as a type of multiple attribute decision making (MADM) problem. By using two different methods in MADM theory, we are measuring the importance levels of quality attributes from different perspectives.

When a number of product alternatives are evaluated by the consumer according to a set of quality attributes, entropy method is most appropriate for application to rank these quality attributes. In entropy method, a quality attribute of a product is treated as an information source from which a certain amount of information, i.e., attribute's importance, is transmitted to the consumer. The second technique, eigenvector method, is applicable for ranking quality attributes from the designer's perspective when the relative importance of quality attributes have been compared in pairs.

If the ranking orders of quality attributes resulting from entropy method and eigenvector method are different, it is necessary to find a compromise between them. This can be done by using an integration model. According to the result from integration, the determinant quality attributes can be identified for directing design efforts.

## Model Development: Multivariate Regression Analysis

When product quality is assumed to be determined by design, the relationship between quality attributes and design factors can be characterized as in Table 3.1.

Table 3.1 Relationship between quality attributes and design factors.

| Quality Attributes | Design Factors |
|---|---|
| Dependent Variables | Independent Variables |
| Responses | Predictors |
| Performance | Task |
| Output | Input |

It should be noted that each design factor may affect different quality attributes to different degrees of significance. To discover how and how much the quality attributes are affected by the specification of design factors, the statistical multivariate analysis is considered an effective tool. Specifically, the relationships between quality attributes and design factors can be modelled by using multivariate regression analysis. In multivariate regression we are interested in predicting several response variables, i.e., quality attributes, from a set of predictors, i.e., design factors, when response variables themselves may be correlated.

Following the procedure of multivariate regression analysis to analyze the data of these two sets of variables, we can formulate a relationship model. By using the model to test with different specifications of design factors, the quality levels of these design alternatives can be predicted. Therefore, the superior design will be identified by comparing the predicted quality levels of design alternatives.

## Quality Measurement: Fuzzy Sets Methodology

"The consumer" is a plural term. Evaluations from consumers must be integrated in order to obtain an objective measurement on quality. This integrated consumer evaluation on each product's quality can be seen as an expected score. For example, when 20% of consumers evaluate one object's quality as "poor", 50% evaluate as "fair", and 30% evaluate as "good", the object's quality score is equal to 0.2*"poor" + 0.5*"fair" + 0.3*"good". However, before the numerical values are assigned to these linguistic scales, the expected score can hardly be calculated.

Conventionally, linguistic scales are mapped to simple numerical grades. Take the above example, suppose the linguistic scales "poor", "fair", and "good" are mapped to the numbers 1, 2, and 3 respectively, the expected score becomes 2.1. Although such mapping is straightforward, and the results are "clean-cut" numbers that may ease further processing task, the uncertain nature associated with these linguistic terms is unduly neglected in the transforming process. Therefore, an improved measurement technique is needed to deal with the uncertainty of perception.

In classical set theory, elements of one set's universe of discourse either belong to a set or do not belong to the particular set. In a similar manner, a statement or solution of traditional scheme for decision making is often stated in the mode of dual logic as true/false, feasible/infeasible, or good/bad. Besides, parameters of a model which represent perceptions are usually assigned with exact numerical values.

However, the comprehension of human thinking and feeling is so imperceptible that the power of dual logic becomes limited. To address this point, the following statement is fit for citation:

> As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior *diminishes* until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" (62).

In science and engineering, real conditions are very often uncertain or vague. These uncertainties can be classified into two categories: (1) random type uncertainty which arises from the system's irregular behavior, and (2) cognitive uncertainty which arises from human thinking, reasoning, cognition and perception processes. Although probability theory is powerful for modeling the randomness of a system, it has limited capability in treating the cognitive vagueness concerning the description of the semantic meaning of the events or phenomena.

The cognitive uncertainty can be related to the consumer perception on products, of which the levels of quality attributes, such as appearance and comfort, can not be naturally and congruously represented in a numerical form. Furthermore, instead of the bivalent judgement on product's quality as either perfect or worthless, consumer's perception is expressed in linguistic terms and shows a graded pattern. Therefore, a method that adopts linguistic approach and allows intermediate assessments is needed. In order to reflect the consumer's perceptions more precisely, the fuzzy sets methodology is applicable since it benefits the analysis by offering a formal treatment of vagueness of natural language concepts.

Fuzzy sets theory was developed by Zadeh (1965) and has become an important tool capable of dealing with a variety of uncertainties including fuzziness, imprecision, and vagueness. In an uncommon manner, fuzzy sets theory provides a strict mathematical framework in which vague conceptual phenomena are precisely and rigorously studied (66). The most succinct definition of fuzzy sets theory has been provided by Zadeh, as:

> ..., the theory of fuzzy sets is, basically, *a theory of graded concepts* - a theory in which everything is a matter of degree or, to put it figuratively, everything has elasticity (63).

Thus, fuzzy sets methodology is essentially concerned with the measurement of imprecise boundaries for variables or sets, such as quality of products.

In contrast with unambiguous statements, a fuzzy statement, which contains the cognitive uncertainty, does not imply precisely demarcated numerical variables but linguistic variables regarding the order of magnitude of attributes of a phenomenon. Unlike the dual logic of classical set theory, the assumption underlying fuzzy sets is that the transition from membership to nonmembership is a gradual but specifiable change. In the following, the linguistic variable and the membership-to-nonmembership transition as a progressional function, i.e. membership function, will be examined with the relevance to the problem of measuring quality.

## Linguistic Variables

Instead of dichotomous grades, conforming or nonconforming, product's quality attributes may be assessed linguistically with gradual levels. This approach is appropriate when (1) the intermediate quality levels need to be assigned, (2) product quality is evaluated subjectively by the consumer, and (3) quality attributes may not be measurable with calibrated instruments.

A linguistic variable, which is defined as a label of a fuzzy set, differs from a numerical variable in that its values are not numbers but words, phrases, or sentences in a natural language. The concept of a linguistic variable serves the purpose of providing a means of approximate characterization of phenomena which are qualitatively uncertain or predominantly subjective in their nature.

According to Zadeh's definition (62), a linguistic or fuzzy variable is characterized by a triple (X, U, R(X;*u*)), in which X is the name of the variable; U is a universe of discourse (finite or infinite set); *u* is a generic name for the elements of U; and R(X;*u*) is a fuzzy subset of U which represents a fuzzy restriction on the values of *u* imposed by X. The assignment equation for X has the form $x = u : R(X)$ and represents an assignment of a value *u* to x subject to the restriction R(X). The degree to which this equation is satisfied will be referred to as the compatibility of *u* with R(X).

A linguistic variable can have one value out of a set of linguistic values. For example, the basic set of linguistic values for the linguistic variable "quality level" may be {terrible, poor, fair, good, excellent}. Each linguistic value can be bound to an interval and restricts a possibility distribution from the numerical universe of discourse. If the range of the universe of discourse is specified as the interval [0, 10], where 0 corresponds to "extremely poor" and 10 to "extremely excellent", the linguistic value "excellent" might correspond to the interval [8, 10], "good" [6, 8], and so on. The assignment of an interval to represent a linguistic value is reasonable because natural language terms are inherently vague, such that the boundary of a term should be a region where the term gradually moves from being applicable to nonapplicable. In addition, the numerical intervals of several linguistic values may overlap due to the existence of inherent fuzziness of words.

## Membership Functions

In fuzzy sets, an object may belong partially to a set. A fuzzy restriction on the numerical universe of discourse, X, is characterized by a membership function, $\mu(x)$, which associates all elements of X into the domain of real numbers defined in the interval from 0 to 1 inclusive, symbolized by [0,1]. That is, the degree of membership is a real number: $0 \leq \mu(x) \leq 1$, where 0 means no membership and 1 means full membership in the set. A particular value of the membership function, such as 0.5, is called a degree of membership.

The relationship between the linguistic variable and the membership function can be formally defined in set theory. If X is a collection of objects denoted generically by x then a

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The relationship between the linguistic variable and the membership function can be formally defined in set theory. If X is a collection of objects denoted generically by x then a fuzzy set Ã in X is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where $\mu_{\tilde{A}}$ is the membership function that maps X to the membership space M, and $\mu_{\tilde{A}}(x)$ is the degree of membership of x in Ã (66). For example, with the assignment of various degrees of membership, a feasible association between a linguistic variable "quality level" and five linguistic values, ranged from "terrible" to "excellent", may be shown in Figure 3.2.

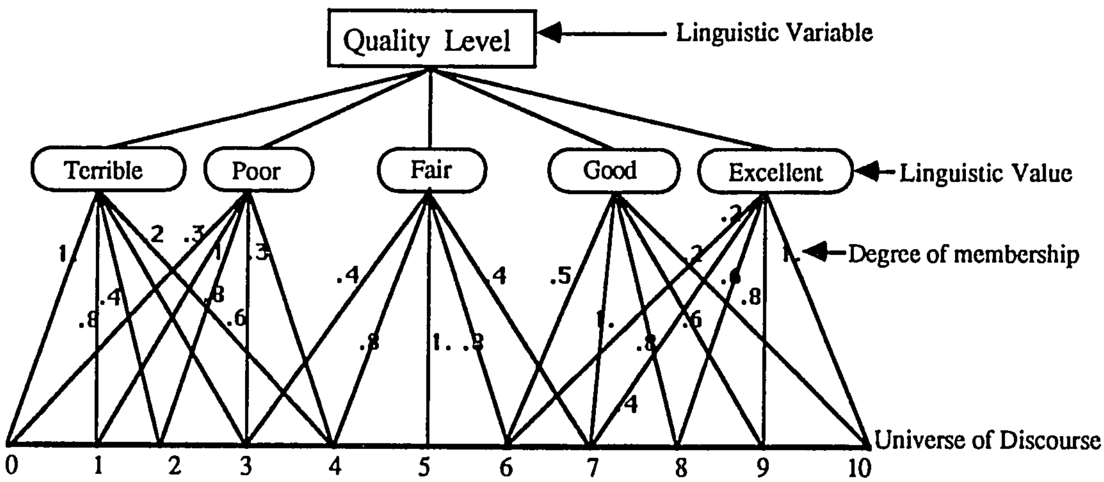


Figure 3.2 A feasible association of a linguistic variable and linguistic values.

It should be noted that the range of universe of discourse is specified according to the nature of the quality attributes. If the quality attributes, such as weight, fuel economy or cost, have the numerical calibration units, such as pound, mile or dollar, corresponding to a linguistic value, the range of universe can be defined based on the conceivable range. For example, while the range of fuel economy (miles/per gallon) of compact cars is [20, 45], the range [35, 45] can be defined for the scale as "excellent". If there is no corresponding calibration unit for the quality attributes, such as comfort or appearance, the range of universe of discourse can be defined as [0, 10], where 0 denotes "extremely terrible" and 10 "extremely excellent".

In this research, the procedures of entropy method, eigenvector method, and multivariate regression analysis are well defined. Throughout these procedures when linguistic values that indicate consumer responses need to be processed, linguistic values will be mapped to fuzzy sets.

## Concluding Remarks

The approach of this research is straightforward. It starts from employing direct magnitude estimation approach to measure consumers' responses to in-market products, followed by the identification of determinant quality attributes and the development of a functional model that describes the relationship between determinant quality attributes and design factors. This relationship model is then used as a working tool for design optimization.

Compared with other methods, such as Quality Function Deployment system, the approach adopted by this research is more comprehensive and direct in relating design with consumer study. Furthermore, since the application methods in this research are supported by rigorous theories, they provide reliable analyses. These methods are synthesized into a complete system and the detailed procedures will be presented in the next chapter.

## CHAPTER IV

## MODELING AND METHODOLOGY

In this chapter, application methods will be examined in pertinent sequence according to the problem solving procedure. This procedure is considered as an integrated method for product design. At the end of this chapter, an example will be presented to illustrate this design method.

### Measurement of Quality: Fuzzy Sets Methodology

As mentioned in the last chapter, consumers evaluate products by using linguistic scales, and fuzzy sets methodology is considered a reliable technique in quantifying these scales. In this research, a method that is based on Monte Carlo simulation technique is implemented to calculate fuzzy numbers.

#### Construction of Membership Functions

The membership function can be determined (1) directly from subjective measurement, (2) by normalizing the results of metric measurements, (3) from a statistical model of the phenomenon, (4) by identification with the fraction of votes for a given assertion, or (5) from a formula or rule chosen from intuitively sound reasoning (33). For this research, a formula with assumed parameter values is used for constructing membership functions.

Suppose a linguistic scale, such as "good", is used to represent a quality level, it can be characterized by a $\pi$-curve function as the scale's membership function. A $\pi$-curve function is a smooth bell shaped curve, symmetrical around a central value (see Figure 4.1). The following formulas define a $\pi$ function, each of them is a piece of the complete curve in a given range (3, 29):

$$\mu(x) = 2\{[x - (a - c)]/c\}^2 \quad \text{if } a - c \le x \le a - c/2$$
$$= 1 - 2[(x - a)/c]^2 \quad \text{if } a - c/2 \le x \le a$$
$$= 1 - 2[(a - x)/c]^2 \quad \text{if } a \le x \le a + c/2$$
$$= 2\{[(a + c) - x]/c\}^2 \quad \text{if } a + c/2 \le x \le a + c. \qquad (4.1)$$

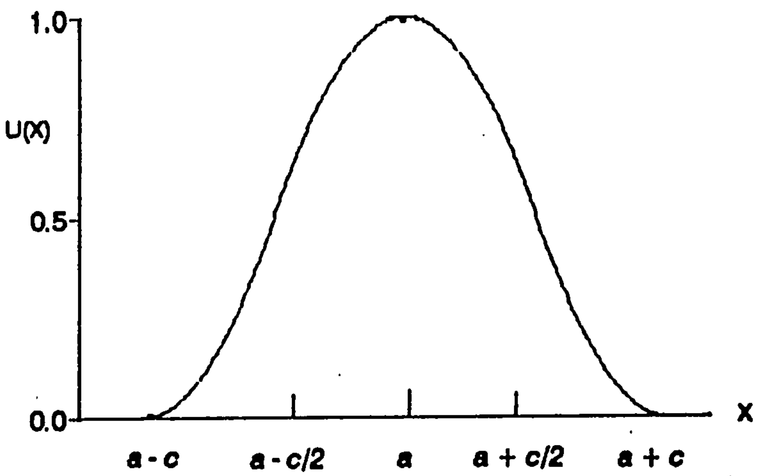


Figure 4.1 The π curve function.

In Equation 4.1, μ(x) is the degree of membership, *a* is the central value, at which the highest degree of membership occurs, and *c* is the range of variation, which reflects the degree of fuzziness. As the degree of fuzziness increases, so will range. The points having null membership are located at $a \pm c$. Except that the values of parameters *a* and *c* can be assumed based on subjective judgement, actual construction of membership functions through an experiment or a survey of opinions may be preferred.

After the values of *a* and *c* are determined with respect to a linguistic scale, a corresponding membership function is defined. Suppose a five-scale scheme is adopted for analysis, the central value of "terrible" may be assumed to be 0, "poor" 2.5, "fair" 5, "good" 7.5, and "excellent" 10 respectively; and the range of variation of all the functions may be assumed to be 2.5. Accordingly, five π-curve membership functions corresponding to these five scales are defined and they are shown pictorially as in Figure 4.2.

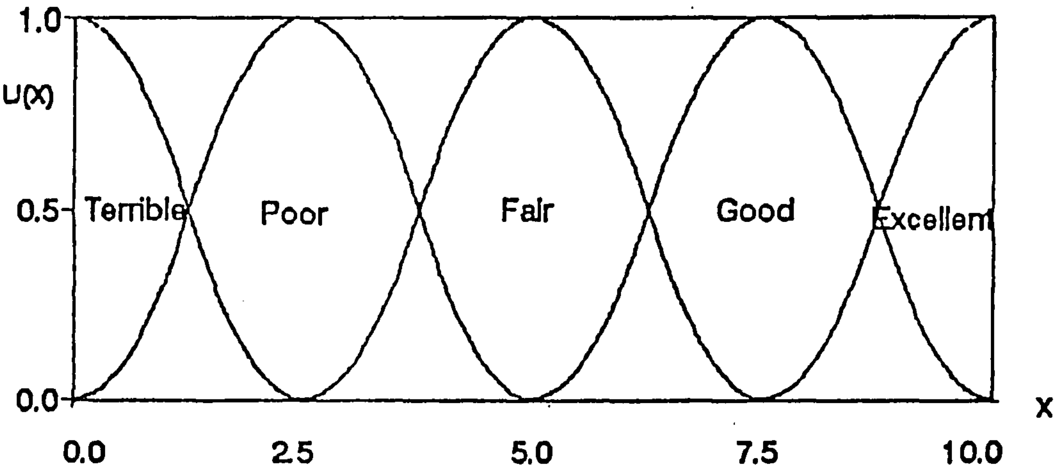


Figure 4.2 π membership functions for five linguistic scales.

## Propagation of Fuzzy Data in Deterministic Systems

As mentioned before, several MADM techniques can be used to rank quality attributes and in-market products, and multivariate regression analysis is applicable to develop a relationship model. Because each application method is well defined, its corresponding algorithm is said deterministic. However, when input parameters or data are fuzzy numbers, a procedure is needed to incorporate with the application method for processing fuzzy numbers. In fuzzy sets, this procedure is called the propagation of cognitive uncertainty or fuzziness in deterministic systems (12).

In the past, the extension principle introduced by Zadeh was used to define mathematical operations such as addition, multiplication, and division to calculate fuzzy numbers in a specified model (62). However, the solution procedure of this approach is very complex. A technique, called vertex method, has been introduced for better efficiency (13). This method basically employs the concept of interval analysis and α-cut representation of fuzzy sets.

Recently, a method based on the Monte-Carlo simulation technique has been developed for computing fuzzy numbers. This new method is called JHE method by its developers (29). As accurate as vertex method, JHE method is even more efficient. Therefore, JHE method, which is presented below as a six-step procedure, is chosen for application in this research.

Step 1. For each membership function, which characterizes a linguistic value that may be assigned to a variable of the deterministic system, determine its cumulative function F(x) by integration. Of each $\pi$-function $\mu(x)$, the corresponding cumulative function is

$$\begin{aligned} F(x) &= [2/(3c^2)]\{x - (a - c)\}^3 && \text{if } a - c \leq x \leq a - c/2 \\ &= (x - a) - [2/(3c^2)](x - a)^3 + (5c)/12 && \text{if } a - c/2 \leq x \leq a \\ &= (x - a) - [2/(3c^2)](x - a)^3 && \text{if } a \leq x \leq a + c/2 \\ &= [2/(3c^2)]\{x - (a + c)\}^3 + c/12 && \text{if } a + c/2 \leq x \leq a + c. \end{aligned} \quad (4.2)$$

Defined as previously, $a$ is the central value and $c$ is the range of variation of the membership function $\mu(x)$. Because each $\pi$ function is not a probability density function, the area under the curve is not equal to 1.0. In fact, the maximum value of the cumulative function F(x), which is the total area under the curve, depends on the range over which the $\pi$-curve is defined.

Step 2. Begin the simulation by generating a uniform random number for each membership function, normalizing it with respect to the maximum functional value of the cumulative function, and then equating the normalized uniform random number to the cumulative function F(x). Then, a value $x$ on the universe of discourse can be back-calculated for the particular membership function. The resulting value $x$ is a random number representing that membership function.

Step 3. Enter the resulting values from Step 2, which are variables' assigned values, into the deterministic system to complete an iteration. Record the output values from the iteration.

Step 4. Repeat Steps 2 and 3 a large number of times.
The number of iterations needed for a converged result may be estimated by trial-and-error procedure.

Step 5. Fit the recorded output values, generated from iterations, into a probability distribution.
In general, the beta distribution function is most proper to fit because:
(1) with given values of the mean, standard deviation, minimum, and maximum calculated from simulation, the derived probability distribution function can be fitted as a beta distribution; and (2) the distribution shape of simulation output may not be symmetric but skewed, while beta distribution may have various shapes with different parameter values.

The beta probability density function is defined over the range $[a,b]$ by

$$f(x) = C\,(x-a)^{\alpha}\,(b-x)^{\beta} \tag{4.3}$$

where $a$ = the minimum value of x,
$b$ = the maximum value of x, and
$C = \{(\alpha + \beta + 1)!\} / \{\alpha!\ \beta!\ (b-a)^{\alpha+\beta+1}\}$.

The shape parameters $\alpha$ and $\beta$ can be obtained from the mean, $\mu$, and the standard deviation, $\sigma$, using the following equations (25):

$$\alpha = X^2(1-X)/Y^2 - (1+X)$$

$$\beta = (\alpha+1)/X - (\alpha+2) \tag{4.4}$$

where $X = (\mu-a)/(b-a)$, and $Y = \sigma/(b-a)$.

Step 6. Normalize the curve-fitted distribution function with respect to its maximum functional value. To normalize, we take the derivative of f(x) with respect to x

$$d/dx\ f(x) = C\,(x-a)^{\alpha-1}\,(b-x)^{\beta-1}\,[-(\alpha+\beta)x + (\alpha b+\beta a)]. \tag{4.5}$$

Taking $d/dx\ f(x) = 0$ yields $x_m = (\alpha b+\beta a) / (\alpha+\beta)$, at which the function f(x) has a maximum value:

$$f(x=x_m) = C\,\alpha^{\alpha}\,\beta^{\beta}\,[(b-a)/(\alpha+\beta)]^{\alpha+\beta}. \tag{4.6}$$

By normalizing f(x) with respect to $f(x_m)$, the desired membership function is obtained:

$$f_N(x) = C_N\,(x-a)^{\alpha}\,(b-x)^{\beta}, \tag{4.7}$$

where $C_N = \{\alpha^{\alpha}\,\beta^{\beta}\,[(b-a)/(\alpha+\beta)]^{\alpha+\beta}\}^{-1}$.

This resulted membership function characterizes an output value from the deterministic system. If there are multiple output variables, the algorithm of the deterministic system will spontaneously yield their resulting values, all in the form of membership function.

To summarize, JHE method starts from defining the cumulative function for each $\pi$-function, generates a normalized uniform random number to equate to the cumulative function, and then back-calculates a value on the universe of discourse. The resulting value is sent to a specified model or system, such as entropy method, to obtain an output value. This procedure is activated for a large number of times, from which a probability distribution of output values is consequently obtained. Lastly, the resulting distribution is normalized into a membership function as the final output. The decision will be made according to the output membership function(s). The procedure of incorporating JHE method with an application method, which is defined as a deterministic system, can be presented as a fuzzy information processing model as in Figure 4.3.

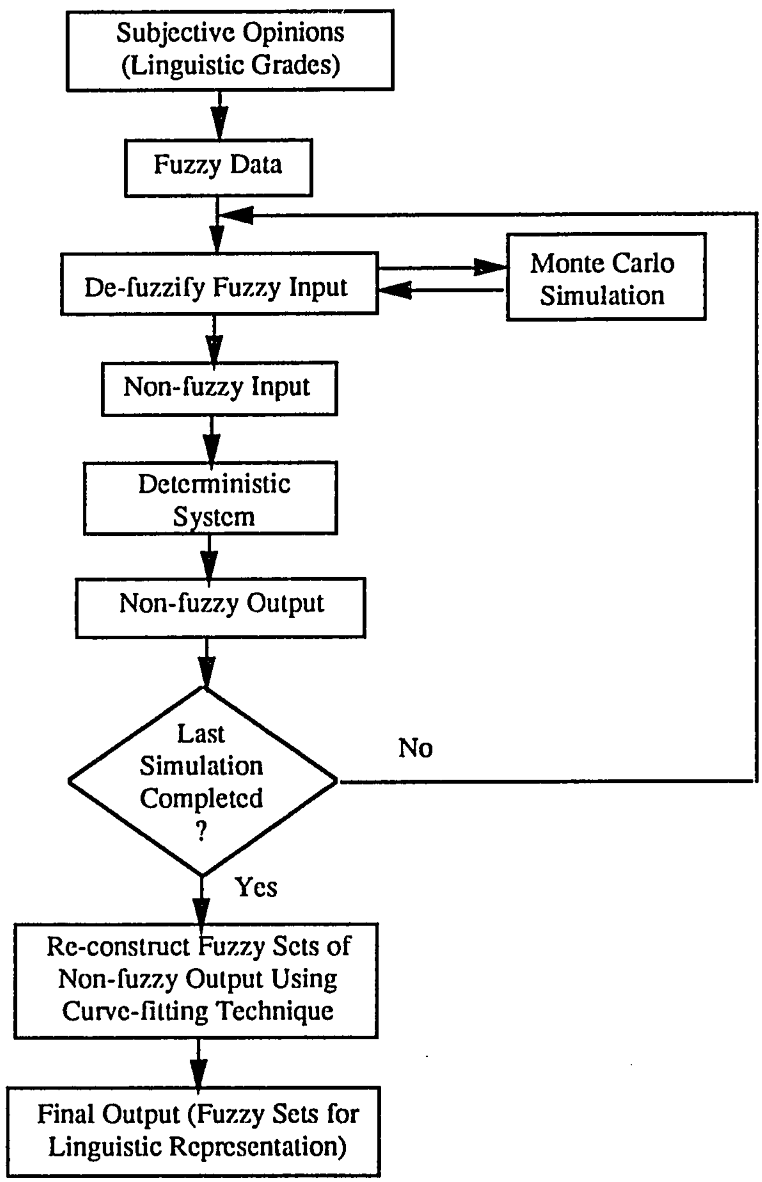


**Figure 4.3 The procedure of JHE method for processing fuzzy data.**

## Attribute and Product Ranking Analysis: MADM Theory

Ranking quality attributes or products can be characterized as a MADM problem, which involves multiple decision makers (consumers as well as the designer), multiple alternatives (products), and multiple attributes or criteria. This section is concerned with the application of efficient and reliable MADM techniques for solving certain problems.

### Attribute Ranking Analysis

To rank quality attributes, entropy method and eigenvector method are applied according to different problem scenarios. If the ranking orders from these two methods are different, an integration model is used to reach a compromise between them. This procedure is shown in Figure 4.4.

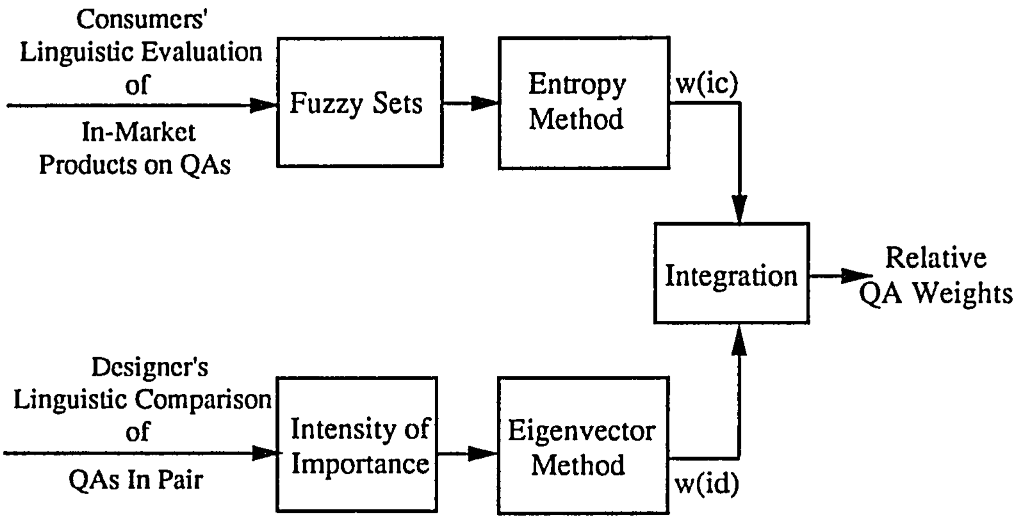


Figure 4.4 Procedure for attribute ranking analysis.

#### Fuzzy Entropy Method: Ranking Quality Attributes from Consumer Perspective

Researches have shown that the assessment of attribute importance can be related to the information concept, and decision making is viewed as an information-processing activity (40). The assumptions of entropy method are (1) decision-relevant information about the available alternatives is transmitted, perceived, and processed via their attributes; (2) the more distinct and differentiated are the scores associated with an attribute (or the larger is the contrast intensity of the attribute), the greater is the amount of "decision information" contained in and transmitted by the attribute; and (3) the more information is transmitted by an attribute, the more salient is the attribute. These assumptions may be supported by marketing researches that people tend to be more sensitive and discerning about the characteristics of greater interest (1).

Since its introduction, the entropy measure of information has been widely used in information science and communication engineering (53). The typical entropy function is defined as $H(X) = - \Sigma \, P(x) \, log \, P(x)$, where $P(x)$ is the probability that the outcome of a random variable X is $x$, and entropy $H(X)$ is the expected amount of received information (48). Entropy method of MADM is developed based on a modified entropy function (64).

Characterized in the form of sets, product alternatives and quality attributes are defined respectively as: $A = \{A_k \mid k = 1,2,...,P\}$, and $Q = \{QA_i \mid i = 1,2,...,M\}$. Suppose a number of consumers evaluate every $A_k$ according to each $QA_i$, a score matrix C can be formed as in Figure 4.5.

$$
C = \begin{array}{c} \\ QA_1 \\ QA_2 \\ \bullet \\ \bullet \\ \bullet \\ QA_M \end{array}
\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \bullet\bullet\bullet & A_P \end{array} \\
\begin{bmatrix} s_{11} & s_{12} & \bullet\bullet\bullet & s_{1P} \\ s_{21} & s_{22} & \bullet\bullet\bullet & s_{2P} \\ \bullet & \bullet & \bullet\bullet\bullet & \bullet \\ \bullet & \bullet & \bullet\bullet\bullet & \bullet \\ \bullet & \bullet & \bullet\bullet\bullet & \bullet \\ s_{M1} & s_{M2} & \bullet\bullet\bullet & s_{MP} \end{bmatrix} \end{array}
$$

Figure 4.5 A sample matrix of attribute scores of products.

Since different consumers may have different perceptions on the quality of product alternatives, $s_{ik}$ is actually an integrated score of $A_k$ according to $QA_i$. The concept of getting expected value of a random variable: $E(X) = \Sigma\, x\, p(x)$ is adopted for calculating $s_{ik}$. Thus, the value of $s_{ik}$ becomes a weighted average of the possible scales:

$$s_{ik} = \sum_{h=1}^{L} s_{ikh}\ p_{ikh}, \quad \text{as } i = 1,\ldots,M \text{ and } k = 1,\ldots,P \tag{4.8}$$

where $s_{ikh}$ is the linguistic scale $h$ of product $k$ on attribute $i$, and $p_{ikh}$ is the percentage of consumers who accredit linguistic scale $h$ to product $k$ on attribute $i$. Totally, there are a number of $L$ linguistic scales, such as five hedonic scales (from "terrible" to "excellent"). Before $s_{ikh}$ is quantified, $s_{ik}$ is still a linguistic variable, while $p_{ikh}$ is a numerical percentage that has been obtained from survey. For example, when the third product alternative is evaluated according to the fourth quality attribute by a group of consumers, 20% assess as "poor", 50% "fair" and 30% "good". In linguistic terms, the value of $s_{43}$ is equal to 0.2 * "poor" + 0.5 * "fair" + 0.3 * "good". To quantify these linguistic scales from fuzzy sets approach, the procedure of JHE method is implemented.

It should be noted that once every $s_{ikh}$ value is generated from a single iteration (see Step 2 of JHE method), a decision matrix **C** is formed and ready to be operated by the entropy method. In other words, the number of **C** matrices is equal to the number of iterations of the simulation procedure. The final outcome from this fuzzy entropy method is the relative weights, presented as membership functions, of quality attributes.

The entropy method actually comprises several related equations (64). The **C** matrix of Figure 4.5 can be characterized in terms of totally $M$ attributes, as a set of vector $s_i = (s_{i1}, s_{i2}, \ldots, s_{iP})$. Let's define $Z_i = \sum_{k=1}^{P} s_{ik}$, $i = 1,\ldots,M$, as the total score regarding the $i$*th* attribute, where $P$ is the number of alternative products. Then the entropy measure of the $i$*th* attribute contrast intensity is

$$e(s_i) = -\tau \sum_{k=1}^{P} (s_{ik}/Z_i)\ ln\ (s_{ik}/Z_i) \tag{4.9}$$

where $\tau$ is a constant, *ln* denotes natural logarithm, and $0 \leq s_{ik}/Z_i \leq 1$. If all $s_{ik}$ becomes identical for a given *i*, then $s_{ik}/Z_i = 1/P$, and $e(s_i)$ assumes its maximum value, that is $e_{max} = ln\ P$. Thus, by setting $\tau = 1/e_{max} = 1/ln\ P$ we achieve $0 \leq e(s_i) \leq 1$ for all $s_i$'s. Such normalization is needed for comparative purposes.

The total entropy of the decision matrix C is defined as

$$E = \sum_{i=1}^{M} e(s_i). \tag{4.10}$$

Observe that the larger $e(s_i)$ is, the less information is transmitted by the ith attribute. Actually, $e(s_i)$ indicates the amount of information *not* transmitted by the *ith* attribute. If $e(s_i) = e_{max} = \ln P$, the *ith* attribute would not transmit any useful information at all. Because attributes' weights, $w_i$, are reversely related to $e(s_i)$, we use $1 - e(s_i)$ to measure the amount of information transmitted by *ith* attribute and normalize to assure that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{M} w_i = 1$:

$$w_i = \frac{1 - e(s_i)}{M - E} . \tag{4.11}$$

It should be noted that every $w_i$ will be presented as a membership function after normalizing the distribution of the resulting numerical $w_i$, and it is interpreted in terms of relative importance (see Steps 3, 4, 5, and 6 of JHE method). Since quality attribute weights are derived from consumer perception on alternative products, any changes in matrix C, regarding alternatives, attributes, or scores, could lead to changes in relative contrast intensities. Consequently such changes get reflected in a new set of $w_i$'s.

The advantages of using entropy method for ranking quality attributes is given below. Firstly, this method does not require individual consumer to rank quality attributes. Secondly, the relative weight of every quality attribute is clearly indicated, so that the attributes of intermediate importance are also detected. Thirdly, the computation of this method is not complex, thus it can be coded into a computer program without difficulty. In addition, in the combination with the fuzzy sets, this method provides linguistic conclusion.

Eigenvector Method: Ranking Quality Attributes from Designer Perspective

Because of one's expertise in design, the relative importance of quality attributes can be compared factually by the designer. Also because the designer should be neutral to in-market products, this attribute ranking method should be product-independent. It is found that eigenvector method is suitable for application since it satisfies these conditions.

Suppose a number of *M* quality attributes ($QA_1$, $QA_2$,..., $QA_M$) are compared in pairs. The set of pair-wise relative importance may be presented as a decision matrix **D** as in Figure 4.6, in which each element $w_{ij}$ is the relative importance of attribute *i* over attribute *j*. Our objective is to recover the weight of each quality attribute, $w_i$, from operating eigenvector method on matrix **D**.

$$
\mathbf{D} = \begin{array}{c} \\ QA_1 \\ QA_2 \\ \bullet \\ \bullet \\ \bullet \\ QA_M \end{array}
\begin{array}{c} \begin{array}{cccccc} QA_1 & QA_2 & \bullet & \bullet & \bullet & QA_M \end{array} \\
\begin{bmatrix} w_{11} & w_{12} & \bullet & \bullet & \bullet & w_{1M} \\ w_{21} & w_{22} & \bullet & \bullet & \bullet & w_{2M} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ w_{M1} & w_{M2} & \bullet & \bullet & \bullet & w_{MM} \end{bmatrix} \end{array}
$$

Figure 4.6 A sample matrix of pair-wise comparisons of attribute relative importance.

Originally, the relative importance of each pair of attributes is presented in linguistic scales. By using a mapping table (Table 4.1), each linguistic intensity scale of comparison can be converted into a numerical scale and **D** becomes a numerical matrix.

Table 4.1 Mapping table for converting intensity scale into numerical scale (taken from 50, 51, and 52).

| Intensity of Importance | Definition | Description |
|---|---|---|
| 1 | Equal importance | Two attributes contribute equally to the objective. |
| 3 | Weak importance of one over another | Experience and judgement slightly favor one attribute over another. |
| 5 | Essential or strong importance | Experience and judgement strongly favor one attribute over another. |
| 7 | Demonstrated importance | An attribute is strongly favored and its dominance demonstrated in practice. |
| 9 | Absolute importance | The evidence favoring one attribute over another is of the highest possible order of affirmation |
| 2, 4, 6, 8 | Intermediate values between the two adjacent judgements | When compromise is needed |
| Reciprocals of above numbers | If attribute i has one of the above numbers assigned to it when compared with attribute j, then j has the reciprocal value when compared with i. | |

Because quality attributes are compared in pairs, the decision matrix **D** is actually a reciprocal matrix, i.e., $w_{ij} = w_i/w_j = 1/w_{ji}$, and $w_{ii} = 1$ when attribute $i$ is compared with itself. The matrix **D** may be formulated as **D w** = $M$ **w**, where **w** is a column vector $(w_1, w_2, \ldots, w_M)^T$. To recover the scale from the matrix of ratios, the nonzero solution may be found to the system **D w** = $M$ w or (**D** - $M$**I**) w = **0**, which is a system of homogeneous linear equations. The system has a nontrivial solution if and only if the determinant of (**D** - $M$**I**) w vanishes, i.e., $M$ is an eigenvalue of **D**. Since every row is a constant multiple of the first row, **D** has unit rank. Thus all the eigenvalues $\lambda_i$, i = 1,...,$M$ of **D** are zero except one, namely $\lambda_{max}$ (the largest eigenvalue of **D**). Corresponding to the $\lambda_{max}$, the nontrivial solution, eigenvector w for the matrix **D** is unique. The solution eigenvector consists of a number of $M$ positive entries. To standardize the eigenvector **w**, each of its entries is normalized by dividing it by entries' sum to obtain the attributes' weights, or in fuzzy sets terminology, the degree of membership of the attributes being compared.

This eigenvector method can be illustrated in a simple example. Suppose a matrix **D** takes the following form:

| | x | y | z |
|---|---|---|---|
| x | 1 | 6 | 4 |
| y | 1/6 | 1 | 1/3 |
| z | 1/4 | 3 | 1 |

where x, y, z are three attributes, and the elements in the matrix are numerical intensities of importance. Follow the power method solving the eigensystem (8), it results $\lambda_{max} = 3.05$, and $w = (0.69, 0.09, 0.22)^T$, in which the entries are the respective normalized weights of attributes x, y and z. It is obvious that the result is consistent with the input data from which the order of the three attributes' weights can be easily recognized.

To summarize, the process begins with a listing of the attributes against themselves in a matrix. A property is chosen and numerical values are assigned according to certain evidence that an attribute reveals the property more than another, thus filling out the matrix. The vector associated with the judgement matrix is known to be the unique solution vector w, of the eigensystem problem. After normalization, the resulting values in the eigenvector are obtained to represent the relative weight of the attributes being compared.

The advantages of applying eigenvector method for ranking quality attributes are: (1) it is natural; that is, it provides a fairly direct translation from the knowledge of qualified observer, i.e., the designer, to the derivation of attribute weights; (2) it results in numerical relative weights of attributes which indicates difference among attributes in terms of importance; and (3) it is easy to compute with, and in many applications, has yielded results which agree accurately with observed data.

Integrating Consumers' and Designer's Quality Attribute Rankings

The priority of attributes perceived by the designer may be different from the priority perceived by consumers. Since neither of the rankings should be overlooked, to derive a compromise between them is a reasonable decision. Accordingly, the weights of each attribute from both ranking procedures are the determinants of importance in parallel fashion such that the most important attribute is always the one having both at their highest levels possible.

Let's denote the weight of attribute *i* from consumer perception as $w_{ic}$, which is presented as a membership function, and the weight from designer perception as $w_{id}$, a membership grade. Both $w_{ic}$ and $w_{id}$ are interpreted in terms of importance. If we assume that the evaluations from consumers and the designer are equally important, the final relative weight of quality attribute i can be formulated as

$$w_i = \frac{w_{ic} \; w_{id}}{\sum_{i=1}^{M} w_{ic} \; w_{id}} \qquad i = 1,\ldots,M. \qquad (4.12)$$

Suppose the significance being set to the evaluations from consumers and the designer are not equal according to some evidence, the analyst's judgement becomes an additional input to the analysis and the above formula is modified to be

$$w_i = \frac{w_{ic}^{\alpha} \; w_{id}^{(1-\alpha)}}{\sum_{i=1}^{M} w_{ic}^{\alpha} \; w_{id}^{(1-\alpha)}} \qquad i = 1,\ldots,M \qquad (4.13)$$

where $\alpha$ and $1-\alpha$ are the respective weighting factors to be assigned to consumers' and the designer's evaluation, with $0 \le \alpha \le 1$. Recognize because that $w_{ic} \subseteq [0, 1]$ and $0 \le w_{id} \le 1$, the greater the value of the exponent $\alpha$ or $1-\alpha$ assigned, the smaller the value of $w_{ic}^{\alpha}$ or $w_{id}^{(1-\alpha)}$

becomes, and less significant the associated characteristic is. Accordingly, a smaller weighing factor should be assigned to a more significant characteristic, e.g., consumer evaluation.

The above formula is a deterministic model which contains fuzzy parameters $w_{ic}$'s, $i = 1,\ldots,M$. Therefore, the procedure of JHE method should be followed to obtain the linguistic conclusion on $w_i$'s. From attribute ranking analysis, the important quality attributes are identified for focusing design efforts and other quality attributes being indicated as less important can be ignored from further analysis.

Product Ranking Analysis

A procedure called "weighted average operation (6)" can be used to rank in-market products. The result from this procedure will indicate which product alternative is most competitive in terms of consumer perceived quality, thus provides supplementary information to develop a competition strategy. The product ranking model is formulated as

$$R_k = \frac{\sum_{k=1}^{P} w_i \; s_{ik}}{\sum_{k=1}^{P} w_i} \qquad (4.14)$$

where $R_k$ is the weighted average weighing of product $k$, $k = 1,\ldots,P$, $s_{ik}$ is the rating of product $k$ with respect to attribute $i$, and $w_i$ is the relative importance of attribute $i$. Remember that the ratings of products are determined by aggregating the perception of a group of consumers, so $s_{ik} = \sum_{h=1}^{L} s_{ikh} \; p_{ikh}$, where $s_{ikh}$ is the linguistic score $h$, $h = 1,\ldots,L$, of product $k$ on attribute $i$, and $p_{ikh}$ is the percentage (probability) of consumers who give linguistic score $h$ as the evaluation to product $k$ on attribute $i$ (Equation 4.8). Also because in-market products are evaluated by consumers, $w_i$ in Equation 4.14 is actually equal to $w_{ic}$.

This "weighted average operation" is another deterministic model, which contains two fuzzy parameters, $w_i$ and $s_{ik}$. Therefore, this model also needs to be incorporated with JHE method to provide linguistic conclusion on weight $R_k$ of in-market product alternatives.

## Model Development: Fuzzy Multivariate Regression Analysis

Having identified the determinant quality attributes, we encounter the problem of modeling the correlation between these quality attributes and a set of design factors. As mentioned in Chapter III, quality attributes are considered as response variables being affected by design factors, which are predictor variables.

For the modeling task, multivariate regression analysis is applicable. The multivariate regression analysis of this research is rather complex because the data associated with response variables is fuzzy, since quality attributes are evaluated in linguistic scales. Therefore, multivariate regression analysis is incorporated with JHE method for model development. This approach can be presented in Figure 4.7.

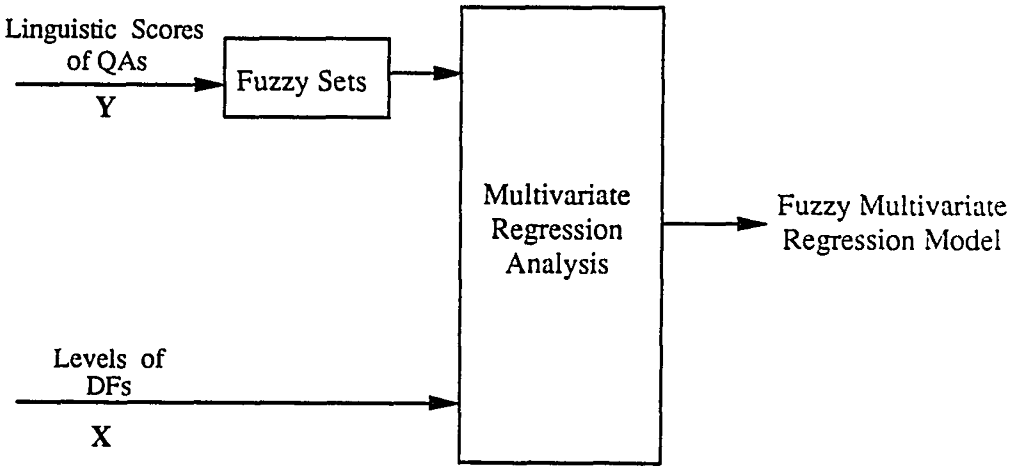


Figure 4.7 Procedure of modeling the correlation between quality attributes and design factors.

Multivariate regression is a generalization of the classical regression models, i.e., univariate regression models. Using multivariate regression analysis, we are modeling multiple response variables simultaneously when response variables themselves may be correlated; and each response variable is modeled as a linear combination of the same set of predictor variables, which may be continuous, categorical, or interaction variables (14). When interaction effect is speculated, original variable(s) can be combined into new variable(s). For multivariate regression, statistical hypotheses concerning the parameters of two or more regression equations, one for each response variable, have to be tested simultaneously. It should be noted that when a model is said to be linear, the linearity is referred in the parameters instead of variables.

For some cases, in which one quality attribute dominates others, the univariate regression analysis is implemented instead. In addition, univariate regression analysis does not address itself to the correlation that exits among response variables. If response variables are independent from each other, using the multivariate linear model is no different from employing univariate model on each response variable individually. No matter the regression analysis is multivariate or univariate, it is used in this study for the following purposes:

1. Variable screening: to detect the degree of significance of each design factor in explaining the variation in quality attribute(s).
2. Model specification: to choose the best model from various candidate models that describe the relationship between a quality attribute or attribute set and design factor set.
3. Prediction: in the use of the specified model to predict the response values, i.e., quality levels of attribute(s), with postulated levels of design factors.

The univariate multiple regression model is based on an *n* x *k* matrix of fixed predictor variables, **X**, and an associated *n*x1 single response variable vector **y**, where *k* is the number of predictor variables, *n* is the number of observations (e.g., product alternatives). In multivariate regression analysis, the model can be extended to include multiple response variable vectors for a given fixed *n* x *k* predictor variable matrix. For example, we might be interested in simultaneously predicting two quality attributes, $y_1$ and $y_2$, from a particular set of design factors, $x_1$, $x_2$ and $x_3$. We regress each quality attribute upon these three design factors:

$$y_1 = \beta_{01} + \sum_{i=1}^{3} \beta_{i1} x_i + \varepsilon_1$$

and

$$y_2 = \beta_{02} + \sum_{i=1}^{3} \beta_{i2} x_i + \varepsilon_2 \ .$$

The errors associated with each equation ($\varepsilon_1$ and $\varepsilon_2$) are assumed to be correlated. Each regression equation has the same values for the predictor variables, but will in general have different regression weights associated with these variables because the relationships between the single set of predictor variables and the response variables will differ. Accordingly, we want to estimate two vectors of regression parameters simultaneously and test various joint hypotheses concerning both sets of regression parameters.

Similar to univariate regression analysis, there are practical issues and assumptions require to be considered in multivariate regression analysis. Although titled with different names, these issues and assumptions discussed below are interrelated.

Causal relationship. The most important limitation to regression analysis concerns inference of causal relationships. Demonstration of causality is a logical and experimental, rather than statistical, problem. Statistics are helpful only in demonstrating that relationships occur reliably. In this research, it is reasonable to assume that consumer perceived quality is related to the specification of design factors. Take car design as an example, the comfort level should be related to the interior space.

Number of observations and variables. Number of variables, including response variables and predictor variables, must be less than the number of sample observations so that error variance can be estimated. If this requirement is violated, we can delete some predictor variables if they are considered less significant based on experience or knowledge, or combine some predictor variables together if possible, or perform a factor analysis of the predictor variables and use factor scores as predictor variables instead of the original predictor variables. By performing this dimension reduction, we can also apply the procedure of eigenvector method of decision making theory to filter out less important design factors.

Outliers. Extreme cases will have deleterious effects on regression solutions, and their influence should be reduced through some legitimate procedures (57).

Multicollinearity and singularity. Calculation of regression coefficients requires inversion of the matrix of correlations among the predictor variables, an inversion that is impossible if predictor variables are singular or completely correlated and is unstable if they are near singular or highly correlated. There are several methods to circumvent this problem, such as simply deleting the least reliable variable, or by transforming the original variables to principle components since they are uncorrelated, or using ridge regression to inflate the variance (57).

Normality, linearity, and homoscedasticity of residuals. The assumption of normality is that the distribution of errors of prediction is independently and normally distributed at all levels of the predicted response variables. Linearity of relationship between predicted response variable scores and errors of prediction is also assumed. The assumption of homoscedasticity is that the standard deviations of errors of prediction are approximately equal at all predicted response variable levels. If failures of theses assumptions are detected, the transformation of variables may be considered to fix the problem (60).

In summary, multivariate regression analysis is used to investigate the mutual dependencies and relationships that exist between sets of response variables **Y**, such as quality attributes, and predictor variables **X**, as design factors. In the Appendix A, the procedure of multivariate regression analysis is presented in detail.

The procedure of multivariate regression analysis is considered as a deterministic model for processing fuzzy data associated with determinant quality attributes. The Equation 4.8 for aggregating consumers' evaluation is used as previously. By implementing JHE method to propagate fuzzy data into regression analysis, we will obtain two primary outputs and both are presented as membership functions: the estimated coefficients of the regression model, **B**, and the statistic Wilks' Lambda, $\Lambda$, which indicates the model's validity and merit. Because $\Lambda$ is presented as a membership function, its mode value is chosen to be evaluated against a "lower percentage **point** of Wilks' Lambda criterion" (60) since there is a maximum degree of membership or confidence associated with the mode.

## Quality Prediction

Once the model is specified, it can be used to predict the utility level of quality attributes with postulated levels of design factors. The prediction can be operated by simply assigning new values for design factors or possibly by using more complex optimization techniques. The latter approach is beyond this research area and needs further study.

Because the estimated coefficients of the regression model are presented as membership functions, we need to propagate fuzzy data into the model to estimate each quality attribute's performance level, which will still be displayed as a membership function. Since this membership function of quality level may not be interpreted directly in linguistic terms, we need to transform it into a numerical value, based on which we can tell the quality level and then make decision upon design specifications. This can be performed by converting a membership function into a utility index by using a mapping model as follows (29):

$$U = (A_l - A_r + 1) / 2 \tag{4.15}$$

where

$U$ = the utility of the membership function (Figure 4.8)

$A_l$ = the area enclosed to the left of the membership function

$A_r$ = the area enclosed to the right of the membership function.

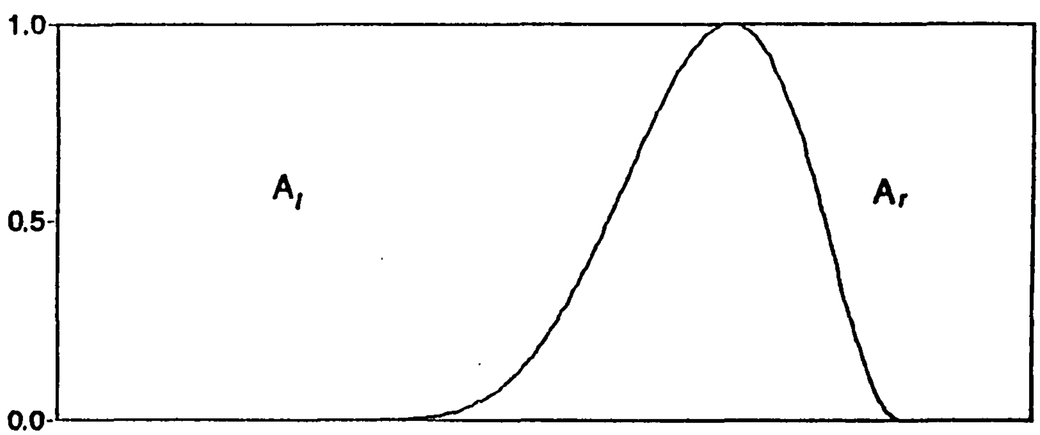


Figure 4.8 A membership function for computing utility index (after Juang et al 1991).

## Computer Programs

According to the developed method, four computer programs, corresponding respectively to each procedure, are written and compiled in FORTRAN on a microcomputer. They are ENTROPY.FOR, EIGEN.FOR, RANK.FOR, and MVREG.FOR. These programs are used in this research to solve an illustrative example in the next section and to validate the method in Chapter V. The program description and computer code for deterministic algorithms are listed in Appendix B.

## Illustrative Example

In this chapter, the design of a tennis racquet is given as an illustrative example to demonstrate the proposed method. The hypothetical scenario is that we, as a tennis manufacturer, are designing a new model of tennis racquet to compete with existing ones, say thirty of them, in the present market.

The input data includes (1) the percentage distribution of testers who evaluate rackets, according to several attributes, by assigning a set of linguistic scales; (2) the attributes' relative importance compared in pairs by the designer; and (3) the specification of a set of common design factors possessed by evaluated rackets. These three data sets can be seen respectively in Appendices C, D, and E. It should be noted that these data are generated for the purpose of illustrating the design method.

According to the Tennis Magazine of March 1990, the set of quality attributes and the set of design factors are shown below.

| Quality Attribute ($QA_i$) | Design Factor ($DF_i$) |
|---|---|
| $QA_1$. Maneuverability | $DF_1$. Hitting area (in sq. in.) |
| $QA_2$. Power | $DF_2$. Beam width (in mm.) |
| $QA_3$. Stiffness | $DF_3$. No. of main strings |
| $QA_4$. Shock damping | $DF_4$. No. of cross strings |
| $QA_5$. Ball control | $DF_5$. Materials |

In this example, the linguistic scales that consumers use to evaluate rackets are prespecified as five-point scales. Corresponding to each of these scales, there is a membership function characterized by assumed parameters (see Table 4.2 and Figure 4.9).

Table 4.2 Parameters of defined membership functions.

| Linguistic Scale | Central Value | Range of Variation |
|---|---|---|
| Excellent | 10.00 | 2.50 |
| Good | 7.50 | 2.50 |
| Fair | 5.00 | 2.50 |
| Poor | 2.50 | 2.50 |
| Terrible | 0.00 | 2.50 |

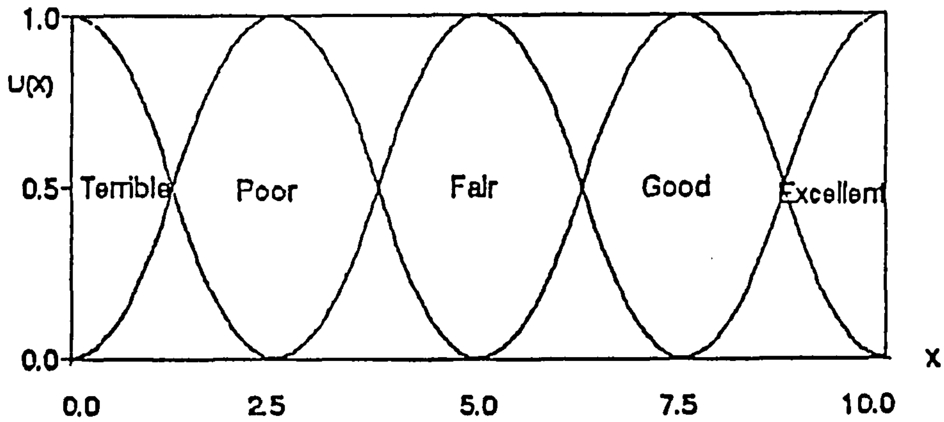


Figure 4.9 Membership functions with specified parameters.

## Quality Attribute Ranking Analysis

### Ranking Quality Attribute from Consumer Perspective

The relative weights of quality attributes perceived by consumers are obtained by using fuzzy entropy method (see Equations 4.8, 4.9, 4.10, 4.11 and JHE method) to operate on the data set in Appendix C. The relative importance of each attribute is presented by a membership function that is characterized by a numerical interval with a mode, which associates with the maximum degree of membership, and shape parameters $\alpha$ and $\beta$ of beta distribution. The result is shown in below table and Figure 4.10.

| | Minimum $w_{ic}$ | Mode $w_{ic}$ | Maximum $w_{ic}$ | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|
| $QA_1$: | [ 0.24 | 0.37 | 0.47 ] | 3.18 | 2.50 |
| $QA_2$: | [ 0.09 | 0.14 | 0.22 ] | 2.13 | 3.04 |
| $QA_3$: | [ 0.11 | 0.17 | 0.26 ] | 2.43 | 3.97 |
| $QA_4$: | [ 0.13 | 0.18 | 0.28 ] | 1.42 | 3.14 |
| $QA_5$: | [ 0.08 | 0.13 | 0.21 ] | 2.50 | 4.03 |

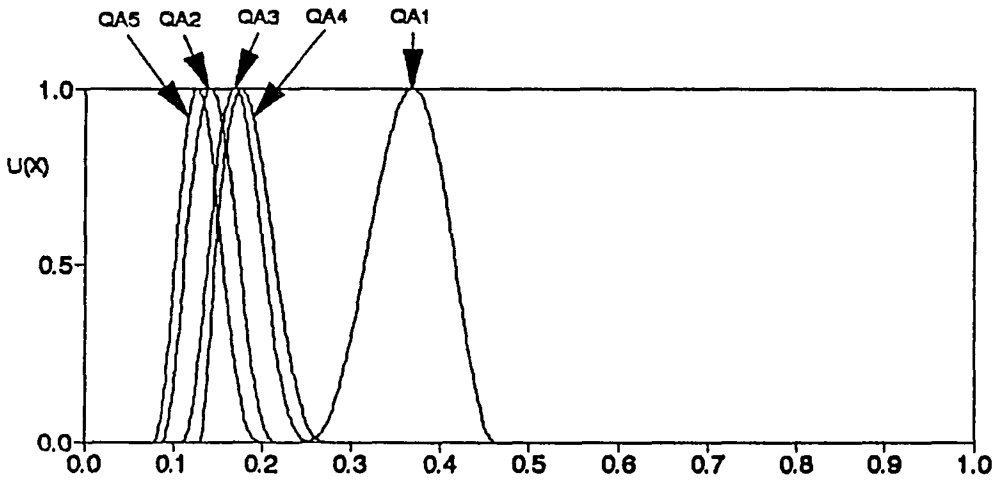


Figure 4.10 Membership functions of attribute weight from consumer perspective.

It is obvious that in terms of relative weight the ranking order of quality attributes from consumer perspective is $QA_1 > QA_4 > QA_3 > QA_2 > QA_5$.

Ranking Quality Attribute from Designer Perspective

The relative importance weight of quality attributes from designer perspective is obtained by using eigenvector method operated on the data set in Appendix D. Each attribute weight is expressed in terms of degree of membership, $0 \leq w_{id} \leq 1$, and because of normalization, $\sum_{i=1}^{5} w_{id} = 1$.

| | $w_{id}$ |
|---|---|
| $QA_1$ | 0.16 |
| $QA_2$ | 0.47 |
| $QA_3$ | 0.23 |
| $QA_4$ | 0.09 |
| $QA_5$ | 0.05 |

The numerical weights indicate attributes' relative importance, therefore the ranking order of quality attributes from designer perspective is $QA_2 > QA_3 > QA_1 > QA_4 > QA_5$. It can be seen that the ranking orders from consumers and the designer are different for this assumed problem.

Ranking Quality Attributes from Integration

Suppose judgements from consumers and the designer are regarded equally significant, the final quality attribute weights are obtained by using the procedure of ranking integration (see Equation 4.12 and JHE method) operated on the output data from entropy method and eigenvector method. As previously, the final weight of each quality attribute is presented as a membership function (Figure 4.11) characterized by a numerical interval and shape parameters α and ß of beta distribution.

| | Minimum $w_{ic}$ | Mode $w_{ic}$ | Maximum $w_{ic}$ | α | ß |
|---|---|---|---|---|---|
| $QA_1$: | [ 0.27 | 0.32 | 0.37 ] | 1.77 | 2.13 |
| $QA_2$: | [ 0.29 | 0.36 | 0.41 ] | 1.71 | 1.06 |
| $QA_3$: | [ 0.16 | 0.21 | 0.24 ] | 2.07 | 1.39 |
| $QA_4$: | [ 0.07 | 0.08 | 0.10 ] | 1.37 | 1.63 |
| $QA_5$: | [ 0.02 | 0.03 | 0.05 ] | 3.68 | 4.23 |

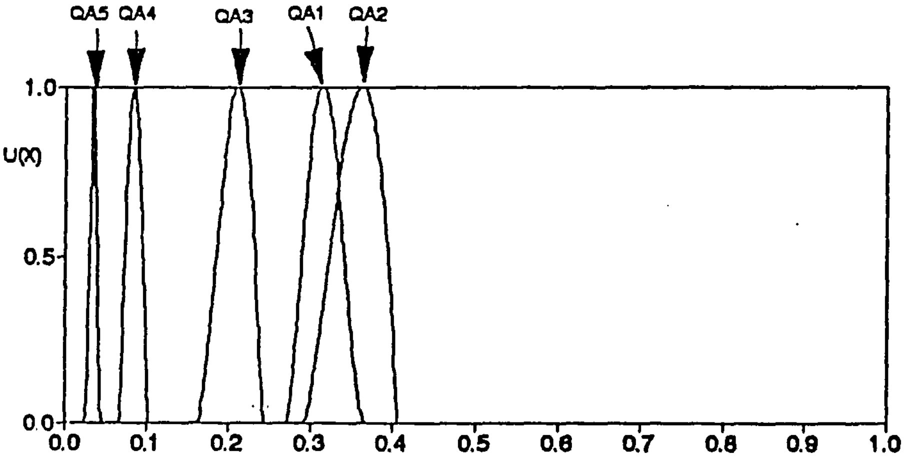


Figure 4.11 Membership functions of final attribute weight.

It can be concluded, then, that the ranking order of quality attributes is $QA_2 > QA_1 > QA_3 > QA_4 > QA_5$. For the concentration of design efforts, we select quality attributes that are considered more important than the others. For the assumed problem, we may select two quality attributes "power" and "maneuverability" as quality niches in designing a tennis racquet.

Development of Fuzzy Multivariate Regression Model

Before proving the racquet's maneuverability, $QA_1$, and power, $QA_2$, are not correlated, we assume they are correlated and model them simultaneously with a set of design factors by using multivariate regression analysis, which is incorporated with JHE method for processing fuzzy parameters. Because the design factor "material" is considered as an indicator variable, which has three distinct categories (graphite, fiberglass, and ceramic), we add two more independent variables, $DF_6$ and $DF_7$, to modeling. Accordingly, values can be assigned to the following variables associated with "material type" are binary as

$$DF_5 = \begin{cases} 1, \text{ if material is graphite} \\ 0, \text{ otherwise} \end{cases}$$

$$DF_6 = \begin{cases} 1, \text{ if material is fiberglass} \\ 0, \text{ otherwise} \end{cases}$$

$$DF_7 = \begin{cases} 1, \text{ if material is ceramic} \\ 0, \text{ otherwise.} \end{cases}$$

Suppose we speculate there is interaction effect between number of main and cross strings of the racquet, we add an eighth design factor, $DF_8$ to the data set. Its value is equal to the number of main strings times the number of cross strings of each alternative rackets.

From several steps of analysis, we find that only two design factors, $DF_1$ and $DF_2$, are significant and the resulting regression model is shown below. As before, each of the model's parameters is presented as a membership function characterized by an interval, at which the degree of membership is the maximum 1, and shape parameters α and ß of beta distribution. Because obtained Wilks' Lambda has a mode value that is less than the critical Lambda, $\Lambda_{critical}(.95; 2,3,27) = 0.626937$, this multivariate regression model is said valid.

$$QA_1 = B_{01} + B_{11}DF_1 + B_{21}DF_2$$

$$QA_2 = B_{02} + B_{12}DF_1 + B_{22}DF_2$$

where

| | Minimum | Mode | Maximum | α | ß |
|---|---|---|---|---|---|
| $B_{01}$: | [ 4.10 | 11.71 | 18.25 ] | 4.47 | 3.84 |
| $B_{11}$: | [ -0.15 | 0.01 | 0.19 ] | 4.08 | 4.47 |
| $B_{21}$: | [ -0.74 | -0.31 | 0.11 ] | 3.83 | 3.79 |
| $B_{02}$: | [ -8.05 | -1.50 | 5.04 ] | 0.89 | 0.89 |
| $B_{12}$: | [ -0.12 | 0.05 | 0.19 ] | 1.11 | 0.90 |
| $B_{22}$: | [ -0.34 | 0.04 | 0.58 ] | 1.20 | 1.71 |
| Λ: | [ 0.001 | 0.002 | 0.008 ] | 2.13 | 7.87. |

If we re-scale the unit of design factor "hitting area" from square inches to square feet, and the unit of the design factor "beam width" from millimeter to one tenth of millimeter, the above model's parameters become

| | Minimum | Mode | Maximum | α | ß |
|---|---|---|---|---|---|
| $B_{01}$: | [ 4.10 | 11.71 | 18.25 ] | 4.47 | 3.84 |
| $B_{11}$: | [ -21.21 | 1.65 | 26.67 ] | 4.08 | 4.47 |
| $B_{21}$: | [ -7.43 | -3.13 | 1.13 ] | 3.83 | 3.79 |
| $B_{02}$: | [ -8.05 | -1.50 | 5.04 ] | 0.89 | 0.89 |
| $B_{12}$: | [-17.50 | 7.46 | 27.63 ] | 1.11 | 0.90 |
| $B_{22}$: | [ -3.40 | 0.40 | 5.80 ] | 1.20 | 1.71 |
| Λ: | [ 0.001 | 0.002 | 0.008 ] | 2.13 | 7.87 . |

It should be recognized that this variable transformation only inflates the coefficients of the former model by the transformation ratio, while the correlation between quality attributes and design factors characterized by the later model remains the same.

## Quality Prediction with Developed Model

Suppose we want to predict the quality levels of a racquet's maneuverability and power with specifications of two design factors: hitting area as 120 square inches and beam width as 30 millimeters. We substitute the specified values into the regression model, then follow JHE method, to obtain the quality levels. These predicted quality levels are presented as below membership functions, each of which is characterized by an interval with a mode and two shape parameters, α and ß. The corresponding utility indices of the attributes' quality levels can also be computed by using equation (4.15).

| | Minimum | Mode | Maximum | α | ß | Utility |
|---|---|---|---|---|---|---|
| $QA_1$: | [ 0.02 | 5.50 | 9.94 ] | 0.52 | 0.22 | 0.17 |
| $QA_2$: | [ 0.11 | 5.59 | 9.96 ] | 0.55 | 1.33 | 1.06 |

Whether the quality levels of "maneuverability" and "power" for this postulated design are acceptable depends on management decision. The procedure for predicting quality and utility can be repeated by assigning other combinations of specifications. By comparing the utility levels from different designs, the optimal and feasible one can be determined.

## CHAPTER V

## METHOD VALIDATION AND DISCUSSION

The method developed in this research for product design is an innovative composition of several analytical methods. Each of these methods has been well developed in a specific field, such as entropy measure in information theory, eigenvector method in decision making theory, and multivariate regression analysis in statistics. However, these methods have never been applied to the problems which are defined in this research. In order to present the soundness of the design method it is important to justify the application of these methods.

Furthermore, it is not a common practice to utilize fuzzy sets methodology for measuring consumer perception to reflect a product's quality level. Therefore, the verification of applying fuzzy sets is a primary subject to address. JHE method, the procedure used in this dissertation for propagating fuzzy data, will be validated by providing its theoretical foundation and proving it can compute "fuzzy numbers" in compliance with the laws of arithmetic. Lastly, the validity of incorporating multivariate regression analysis with fuzzy sets will be presented by examining the robustness of the developed regression model.

### Validation of Entropy Method

Entropy method of measuring the relative importance of attributes is based on entropy measure of information. Although the derivation of entropy method by Zeleny (64) is coherent and convincing, the rationale behind it should be described more explicitly.

There are two implicit axioms in ranking a group of attributes possessed by certain object(s): (1) the degree of distinctions of the scores associated with an attribute reflects the amount of information transmitted by this particular attribute; (2) an attribute's degree of importance is measured by the proportion of information it contributes to the total amount of information transmitted by all the attributes. Because entropy method is modeled based on these two axioms, its robustness should be tested against them.

The main operation of entropy method and its validity is focused upon the measure of an attribute's contrast intensity (Equation 4.9)

$$e(s_i) = -\tau \sum_{k=1}^{P} (s_{ik}/Z_i) \ln (s_{ik}/Z_i), \quad i = 1 \text{ to } M.$$

This measure is derived from the entropy measure $H(X) = -\Sigma P(x) \log P(x)$. In information theory, the quantity of $H(X)$ can be interpreted as average amount of surprise, uncertainty, or information received when the value of the random variable X is observed (48). In Equation 4.9, the entropy being measured involves a normalization not used in the common definition, and it is interpreted as the amount of information *not* being contained in and transmitted by an attribute. In other words, the amount of information transmitted by the *ith* attribute is reversely related to $e(s_i)$. This way of unusual interpretation is rationalized as follows.

To measure an attribute's contrast intensity, a constant coefficient $\tau$, which is equal to $1/e_{max}$ or simply $1/(\ln P)$, is assigned to the entropy measure, where $e_{max}$ is the maximum value that $e(s_i)$ may assume, and $P$ is the number of alternatives. It is observed that only when all the scores, $s_{ik}$, associated with attribute $i$ are identical,

$$\tau = -1 / \{ \sum_{k=1}^{P} (s_{ik}/Z_i) \ln(s_{ik}/Z_i)\} = 1/(\ln P)$$

and $e(s_i)$ is assumes its maximum, 1.

Because the amount of information *not* transmitted by attribute $i$ is measured by $e(s_i)$, we use $1 - e(s_i)$ as the measure of the amount of transmitted information, which is defined as the "degree of diversification." If $e(s_i) = 1$, the degree of diversification of *ith* attribute becomes 0, indicating that no useful information is transmitted. Otherwise, a certain amount of information transmitted by *ith* attribute will be reflected due to the variation of scores.

It is also noticed that the greater $P$ is the smaller the constant $\tau$. Further, both $\tau$ and $e(s_i)$ can not be calculated if $P$ is 1; only when $P$ is equal to or greater than 2, entropy $e(s_i)$ is measurable. Therefore, it is reasonable to apply entropy method for ranking quality attributes from consumer perspective since consumer preference is revealed when multiple product alternatives are available for evaluation.

The total entropy of decision matrix C is the sum of $e(s_i)$, that is (Equation 4.10)

$$E = \sum_{i=1}^{M} e(s_i) \quad .$$

$E$ is defined as the measure of the total amount of information *not* transmitted by $M$ attributes. Then the total amount of information transmitted by $M$ attributes is measured by the sum of $1 - e(s_i)$, or equivalently $M - E$. Also because an attribute's relative weight is considered as the proportion of amount of information it contributes to the total amount of information transmitted by all the attributes, that attribute's weight can be calculated by Equation 4.11:

$$w_i = \frac{1 - e(s_i)}{M - E} \qquad i = 1 \text{ to } M \quad .$$

Because $0 \le e(s_i) \le 1$, therefore $0 \le E \le M$. There are two extreme cases that rarely occur in ranking attributes: (1) every attribute transmits information to the outermost extent that every $e(s_i)$ as well as $E$ become 0, then all attributes are found equally important, i.e., $w_i = 1 / M$; and (2) no information is transmitted by any of the attributes, i.e., every $e(s_i)$ is equal to 1 and $E$ is equal to $M$, then attribute weight can not be calculated. Normal cases that fall between these two extremes can be analyzed to calculate the attribute weight.

In the following, numerical examples are presented to illuminate the above discussion, thus enhance the method's validity. We first use the simple example given in Zeleny's "Multiple Criteria Decision Making" (page 194), then modify this problem into several cases to test the method's robustness. Consider that three common attributes, $Q_i$ as i = 1 to 3, are associated with four alternatives, $A_k$ as k = 1 to 4. The measured scores are summarized in the below matrix.

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $Q_1$ | 7 | 8 | 8.5 | 9 |
| $Q_2$ | 100 | 60 | 20 | 80 |
| $Q_3$ | 4 | 4 | 6 | 2 |

Following the procedure of entropy method, we obtain the entropy measure, $e(A_i)$, and relative weight, $w(Q_i)$, of three attributes as:

$$\begin{array}{ll} e(Q_1) = 0.997 & w(Q_1) = 0.022 \\ e(Q_2) = 0.913 & w(Q_2) = 0.640 \\ \underline{e(Q_3) = 0.954} & \underline{w(Q_3) = 0.338} \\ E \;\; = 2.864 & \Sigma\, w(Q_i) = 1.000 \end{array}.$$

Suppose an additional attribute, $Q_4$, is added to the decision matrix, we have following four feasible cases for testing entropy method.

Case 1. Suppose all the scores associated with $Q_4$ are identical. For example, $s_{4k} = 32$ as $k = 1$ to 4. Following the procedure of entropy method, we obtain the entropy measure, $e(Q_i)$, and attribute weight, $w(Q_i)$, as below.

$$\begin{array}{ll} e(Q_1) = 0.997 & w(Q_1) = 0.022 \\ e(Q_2) = 0.913 & w(Q_2) = 0.640 \\ e(Q_3) = 0.954 & w(Q_3) = 0.338 \\ \underline{e(Q_4) = 1.000} & w(Q_4) = 0.000 \\ E \;\; = 3.864 & \end{array}$$

This result shows that when the scores associated with an attribute are identical, that attribute does not transmit any useful information, therefore it has no significance at all. Furthermore, the addition of this new attribute does not affect the conclusion made on other attributes.

Case 2. Suppose attribute $Q_4$ has fairly small variation in scores. For example, $s_{41} = 31$, and $s_{4k} = 32$ as $k = 2$ to 4. We obtain the entropy measure, $e(Q_i)$, and attribute weight, $w(Q_i)$, as below.

$$\begin{array}{ll} e(Q_1) = 0.997 & w(Q_1) = 0.022 \\ e(Q_2) = 0.913 & w(Q_2) = 0.633 \\ e(Q_3) = 0.954 & w(Q_3) = 0.334 \\ \underline{e(Q_4) = 0.999} & w(Q_4) = 0.005 \\ E \;\; = 3.862 & \end{array}$$

This result shows that an attribute with small variation in scores transmits negligible amount of information, hence it is trivial.

Case 3. Suppose scores associated with $Q_4$ are highly distinct from each other, such as $s_{41} = 1$, $s_{42} = 8$, $s_{43} = 16$, and $s_{44} = 32$. We obtain the entropy measure, $e(Q_i)$, and attribute weight, $w(Q_i)$, as below.

| | |
|---|---|
| $e(Q_1) = 0.997$ | $w(Q_1) = 0.008$ |
| $e(Q_2) = 0.913$ | $w(Q_2) = 0.219$ |
| $e(Q_3) = 0.954$ | $w(Q_3) = 0.119$ |
| $e(Q_4) = 0.741$ | $w(Q_4) = 0.654$ |
| $E = 3.604$ | |

This result shows that if an attribute has high variation in scores it transmits great amount of information, therefore is highly significant.

Case 4. Suppose scores associated with $Q_4$ are identical to the scores associated with $Q_2$, that is $s_{41} = s_{21} = 100$, $s_{42} = s_{22} = 60$, $s_{43} = s_{23} = 20$, and $s_{44} = s_{24} = 80$. We obtain the entropy measure, $e(Q_i)$, and attribute weight, $w(Q_i)$, as below.

| | |
|---|---|
| $e(Q_1) = 0.997$ | $w(Q_1) = 0.014$ |
| $e(Q_2) = 0.913$ | $w(Q_2) = 0.388$ |
| $e(Q_3) = 0.954$ | $w(Q_3) = 0.211$ |
| $e(Q_4) = 0.913$ | $w(Q_4) = 0.388$ |
| $E = 3.776$ | |

This result shows that two attributes with the same variation in scores will transmit equal amount of information, therefore are equally important.

Without incorporating entropy method with JHE method, we may use entropy method alone to rank the five quality attributes associated with thirty tennis rackets from consumer perspective (also see Illustrative Example of Chapter IV). Instead of considering the fuzziness of linguistic scales, we only use the central point to represent each of five scales. We obtain the result as follows :

| $QA_i$ | $w_r(QA_i)$ | $Rank(QA_i)$ |
|---|---|---|
| 1 | 0.38 | 1 |
| 2 | 0.15 | 4 |
| 3 | 0.16 | 3 |
| 4 | 0.19 | 2 |
| 5 | 0.12 | 5 |

This result is consistent with the ranking order of using fuzzy entropy method. Furthermore, the numerical weight of each quality attribute falls within the range of membership function of attribute weight, at the same time it is fairly close to the mode of membership function. At this point, we have not only confirmed entropy method's robustness but also gained confidence in fuzzy entropy method.

## Validation of Eigenvector Method

Eigenvector method is applied to ascertain attribute weight from the designer's perspective. In the matrix of pair-wise comparisons of attributes' relative weight, each element, $w_{ij}$, is actually equal to $w_i/w_j$. Also because the matrix is reciprocal, $w_{ji}$ is equal to $1/w_{ij}$, or equivalently $w_j/w_i$. Once the values of $w_i$ and $w_j$ are obtained from the procedure of eigenvector method, we can rebuild a new matrix that consists of the ratios of $w_i$ and $w_j$. When we repeat the operation of eigenvector method on the new matrix to calculate attributes' weight, we expect the result to be identical to the result from initial matrix. If both results are indeed the same, eigenvector method is affirmed to be effective in recovering attribute's hidden weight from a pair-wise comparison matrix.

To validate eigenvector method, we are actually confirming its effectiveness and we consider showing numerical examples is appropriate for this purpose. Firstly, we use the example given in Saaty's "The Logic of Priorities" (page 27) to demonstrate eigenvector method. Suppose there are four attributes that a person may possess to characterize one's success: hard work (HW), productivity (PR), intelligence (IN), and perseverance (PE). The result of attribute weight from operating eigenvector method on the given decision matrix is: $w_{HW} = 0.06$, $w_{PR} = 0.55$, $w_{IN} = 0.17$, and $w_{PE} = 0.22$. Assuming these weights are the true attribute weights, a new matrix that consists of the ratios of these weights is built as

| | HW | PR | IN | PE |
|---|---|---|---|---|
| HW | 1 | 0.11 | 0.36 | 0.29 |
| PR | 8.87 | 1 | 3.18 | 2.56 |
| IN | 2.79 | 0.32 | 1 | 0.81 |
| PE | 3.47 | 0.39 | 1.24 | 1 |

Then we repeat the procedure of eigenvector method to obtain a vector of attribute weight as $w = (0.06, 0.55, 0.17, 0.22)^T$, which is indeed identical to the one that has been assumed true. Accordingly eigenvector method's effectiveness is confirmed.

Here we adopt the same approach to test eigenvector method once more by using the example given in the Chapter IV of this dissertation. By using eigenvector method we ranked racket's five quality attributes from the designer's perspective, and obtained the vector of attributes weights as $w_d = (w_1,w_2,w_3,w_4,w_5)^T = (0.16, 0.47, 0.23, 0.09, 0.05)^T$, which is assumed to be authentic. Based on this vector of attribute weights, we rebuild the matrix of pair-wise comparisons of attributes into a new matrix as

| | $QA_1$ | $QA_2$ | $QA_3$ | $QA_4$ | $QA_5$ |
|---|---|---|---|---|---|
| $QA_1$ | 1 | 0.33 | 0.69 | 1.84 | 3.12 |
| $QA_2$ | 3.00 | 1 | 2.08 | 5.53 | 9.37 |
| $QA_3$ | 1.45 | 0.48 | 1 | 2.66 | 4.51 |
| $QA_4$ | 0.54 | 0.18 | 0.38 | 1 | 1.70 |
| $QA_5$ | 0.32 | 0.11 | 0.22 | 0.59 | 1 . |

Following the procedure of eigenvector method operated on the above matrix, we obtain relative weights and ranks of quality attributes as

| $QA_i$ | $w_d(QA_i)$ | $Rank(QA_i)$ |
|---|---|---|
| 1 | 0.16 | 3 |
| 2 | 0.47 | 1 |
| 3 | 0.23 | 2 |
| 4 | 0.09 | 4 |
| 5 | 0.05 | 5 . |

This result is identical to the input $w_d$ vector, and it again shows that the actual weights of attributes can be obtained when the pair-wise comparison matrix is operated by the eigenvector method.

## Validation of Ranking Integration Procedure

Once results are obtained from entropy method and eigenvector method respectively, we wish to determine quality attributes' final weights and ranks as the compromise between consumers and the designer. A simple formula (Equation 4.12) is used to integrate the two results:

$$w_i = \frac{w_{ic} \; w_{id}}{\sum_{i=1}^{M} w_{ic} \; w_{id}} \qquad i = 1,\dots,M.$$

According to this formula, the two weights, $w_{ic}$ and $w_{id}$, of quality attribute $i$ counterbalance or offset each other, therefore we insure the most important attributes are the ones having both at their highest levels possible. Take "tennis racket" as an example, there are five quality attributes, each of which has two types of weight with associated ranking order: $w_{ic}$ / $R_c$ and $w_{id}$ / $R_d$. Using Equation 4.12, we can determine final quality attribute weights and ranks (last two columns of below table).

| $QA_i$ | $w_{ic}$ | $R_c$ | $w_{id}$ | $R_d$ | $w_i$ | R |
|---|---|---|---|---|---|---|
| 1 | 0.38 | 1 | 0.16 | 3 | 0.32 | 2 |
| 2 | 0.15 | 4 | 0.47 | 1 | 0.37 | 1 |
| 3 | 0.16 | 3 | 0.23 | 2 | 0.19 | 3 |
| 4 | 0.19 | 2 | 0.09 | 4 | 0.09 | 4 |
| 5 | 0.12 | 5 | 0.05 | 5 | 0.03 | 5 |

The above result is consistent with the conclusion of the example shown in Chapter IV. If the integration model is not used, it is difficult to make conclusion merely by examining attributes' ordinal ranks.

## Validation of JHE Method

The notion of fuzzy data propagation is to cope with fuzzy data effectively and efficiently so that they can be operated by a deterministic model. These fuzzy data or parameters denote subjective perceptions and they are presented in the form of membership functions, or called fuzzy numbers, instead of real numbers. For example, "good" is a perception scale which may coexist with other scales, such as "terrible", "poor", "fair", and "excellent". Along a real line ranged from 0 to 10, the numerical indication of "good" may be "about 7.5". In a sense, "good" is represented by 7.5 with highest confidence level, while the confidence level decreases when other numbers are used to indicate "good".

Since only numbers can be operated directly by a model, we have to take the function one "bit" a time to complete a calculation. The inverse transform technique is applicable to meet this need. The inverse transform technique is utilized when the inverse of a cumulative distribution function (cdf) can be explicitly computed analytically, and it is summarized as following steps (4):

Step 1. Compute the cdf of the desired random variable $X$: $F(x) = \int_{-\infty}^{x} f(t)\, dt$.

Step 2. Set $F(X) = R$ on the range of $X$, $x \geq 0$. $R$ has a uniform distribution over the interval [0, 1].

Step 3. Solve the equation $F(X) = R$ for $X$ in terms of $R$. As a random variate generator for a distribution, $X = F^{-1}(R)$.

Step 4. Generate uniform random numbers $R_1$, $R_2$, $R_3$,... and compute the desired random variates by $X_i = F^{-1}(R_i)$.

JHE method for fuzzy data propagation is actually based on the inverse transform technique with additional steps: (1) advance the obtained random number to a model to result in an output; (2) repeat the same procedure for a great number of times to assure the entire fuzzy number(s) is operated by a deterministic model; and (3) fit the output values generated from iterations into a probability distribution, which is then normalized into a desired membership function(s). In this manner, fuzzy numbers can be directly computed.

As real numbers, fuzzy numbers should be computable in compliance with the laws of arithmetic. If these laws indeed apply to fuzzy numbers when a propagation procedure operates, the propagation procedure itself is thus validated. In this research, JHE method is the propagation procedure of concern. In the following, each of these laws will be specified with fuzzy numbers being computed by a simple operation, such as addition or multiplication. According to a rule, e.g., $a + b = b + a$ of commutative laws, an output value resulted from the right-hand-side equation will be compared with the output value from the left-hand-side equation, whereas both outputs are presented as fuzzy numbers. If the results from both sides are identical or have negligible difference, the associated rule is said to be applicable to the input fuzzy numbers, and accordingly JHE method is valid.

Because the output values generated from iterations are fitted into a beta distribution (see Step 5 of JHE method) for constructing a fuzzy number, that fuzzy number is characterized by an interval, a mode value, and two shape parameters of beta distribution. In the following examples, both input and output fuzzy numbers are presented in the form of $[F_{min}, F_{mode}, F_{max}]$; where $F_{min}$ and $F_{max}$ are the lower bound and upper bound, and $F_{mode}$ is the mode value.

1. Commutative Laws. $a + b = b + a$, and $ab = ba$.

Example. $a = [0, 2.5, 5]$, $b = [2.5, 5, 7.5]$

$$a + b = [2.7, 7.6, 11.9] = b + a;$$

$$ab = [0.3, 10.8, 33.8] = ba.$$

2. Associative Laws. $(a + b) + c = a + (b + c)$, and $(ab)c = a(bc)$.

Example. $a = [0, 2.5, 5]$, $b = [2.5, 5, 7.5]$, $c = [5, 7.5, 10]$

$$(a + b) + c = [7.8, 14.9, 21.6] = a + (b + c);$$

$$(ab)c = [1.4, 45.4, 327.8] \approx a(bc) = [1.4, 46.4, 327.8].$$

3. Distributive Laws. $a(b + c) = ab + ac$

Example. $a = [0, 2.5, 5]$, $b = [2.5, 5, 7.5]$, $c = [5, 7.5, 10]$

$$a(b + c) = [0.8, 25.6, 79.4] \approx ab + ac = [0.8, 25.5, 79.3].$$

4. Identity Laws. $a + 0 = 0 + a$, and $a \cdot 1 = 1 \cdot a$.

Example. $a = [0, 2.5, 5]$, $0 = [0, 0, 0]$, $1 = [1, 1, 1]$

$$a + 0 = [0.1, 2.5, 4.9] = 0 + a;$$

$$a \cdot 1 = [0.1, 2.5, 4.9] = 1 \cdot a.$$

5. Inverse Laws. $a + (-a) = (-a) + a = 0$, and $aa^{-1} = a^{-1}a = 1$.

Example. $a = [0, 2.5, 5]$

$$a + (-a) = [0, 0, 0] = (-a) + a;$$

$$aa^{-1} = [1, 1, 1] = a^{-1}a.$$

6. Rules for Quotients. $(a/b) + (c/d) = (ad + bc)/(bd)$,

$$(a/b)(c/d) = (ac)/(bd),$$

$$(a/b)/(c/d) = (ad)/(bc).$$

Example. $a = [0, 2.5, 5]$, $b = [2.5, 5, 7.5]$, $c = [2.5, 5, 7.5]$, $d = [5, 7.5, 10]$

$$(a/b) + (c/d) = [0.6, 1.2, 1.4] = (ad + bc)/(bd),$$

$$(a/b)(c/d) = [0.0, 0.4, 0.5] = (ac)/(bd),$$

$$(a/b)/(c/d) = [0.1, 0.8, 0.9] = (ad)/(bc).$$

It should be noted that the accuracy of the output fuzzy numbers is proportional to the number of iterations in propagation procedure; the greater the number of iterations, the more accurate the result. For all the above examples, the iteration number is set to be 2000. In some examples, negligible difference exists between the outputs of right hand side and left hand side. It is expected that the disparity would be reduced with more iterations. The examples also show that the membership functions of input fuzzy numbers may be designed as symmetrical, but the membership functions of output fuzzy numbers are likely to be skewed.

At this point, we conclude that fuzzy numbers indeed comply with the laws of arithmetic. In other words, the validity of JHE method is confirmed. As mentioned before, fuzzy sets method has the advantage over numerical mapping scheme in presenting qualitative data, JHE method provides an effective tool to enhance the capability in data analysis and decision making.

## Validation of Fuzzy Multivariate Regression Analysis

Multivariate regression analysis is applied in this research to model the correlation between important quality attributes and design factors. Since multivariate regression analysis is a statistical tool that has been well developed, its applicability is more concerned than its theoretical validity in this research.

As mentioned in Chapter IV, there are several practical issues required to be considered in applying multivariate regression analysis to an actual problem. Moreover, we are especially concerned if the incorporation of multivariate regression analysis and fuzzy sets, or called fuzzy multivariate regression analysis, will generate a proper regression model. In the following, the validity and advantage of fuzzy multivariate regression analysis will be demonstrated.

An analytical method or procedure can be viewed as an input-output transformation, that is, the method accepts values of the input data and transforms these inputs into output measures of performance. If two or more methods transform the same input and the obtained outputs are consistent, we can conclude with confidence that these methods are agreeable.

Further more, if one of the methods is valid, the others' validity can be accepted or rejected by contrasting their outputs with the output from the valid method. In this section, this concept of input-output transformation is adopted for validating the procedure of multivariate regression analysis with and without incorporating with fuzzy sets. Input data is the evaluation distribution of consumers as listed in Appendix C, and output of the multivariate regression analysis is the regression model.

The procedure of multivariate regression analysis is treated as a deterministic system in the propagation procedure. Same as the discussion in validating entropy method, besides fuzzy input parameters, multivariate regression analysis can operate with non-fuzzy parameters as well. In other words, with the same data sets, multivariate regression analysis can operate with fuzzy and non-fuzzy parameters and generate fuzzy and non-fuzzy multivariate regression models accordingly. However, except the consistency between fuzzy and non-fuzzy models can be confirmed or denied, it is difficult to judge which is the valid or better model by directly comparing two models. Therefore we adopt an indirect approach for validation by assuming a "true" multivariate regression model, which relates quality attributes and design factors. With the true model we can generate data for the value of quality attributes by plugging the pre-determined value of design factors into the model. With the data sets of estimated value of quality attributes and determined value of design factors, we can implement multivariate regression analysis with and without incorporating fuzzy sets to develop fuzzy and non-fuzzy models. By comparing the obtained models, fuzzy and non-fuzzy, with the true model, we can distinguish if multivariate regression analysis incorporating with fuzzy sets is a better method.

In brief, we try to restore the true model by using the data generated from the true model, and the objectives are twofold: (1) demonstrate the consistency between generated models, both fuzzy and non-fuzzy, and the true model; and (2) demonstrate the advantage of fuzzy multivariate regression analysis for model development. These objectives can be accomplished by generating different data sets from the true model.

It should be noted that the data generated from the true model is the percentage of consumer evaluation, and the evaluation distribution of consumers may not result in expected score of an alternative product that is identical with the true score. The distortion of evaluation distribution of consumers is due to the reality that some consumers may not use the proper scales for evaluation, therefore the true score of an alternative may not exactly be reflected from consumer evaluation. For example, some consumers may evaluate a product's quality attribute as fair while actually it's good, and thus distort the true distribution. In such a case, fuzzy sets is reliable since it takes care of the overlaps, the fuzzy parts, among qualitative scales.

Therefore for the purpose of validation, we can deliberately distort the distribution of consumer evaluation to a certain degree, thus deviate the expected score from the true score, in order to examine whether fuzzy or non-fuzzy multivariate regression model will be consistent with the true model. It is expected that when the distribution is slightly distorted, then the expected score should be very close to the true score, and both the fuzzy and non-fuzzy models will be agreeable with the true model. Until the distortion reaches a certain degree, only the fuzzy model will be consistent with the true model while the non-fuzzy model fails. For generating data sets on the percentage of consumer evaluation with respect to different degrees of distortion, a procedure listed in Appendix F can be followed.

Still using the example of tennis racket design, we assume the true model to be

$$QA_1 = 10 + 6\ DF_1 - 3\ DF_2$$

$$QA_2 = -3 + 8\ DF_1 + 1\ DF_2$$

where $QA_1$ is the attribute "maneuverability", $QA_2$ is the attribute "power", $DF_1$ is the design factor "hitting area" (in sq. ft.), and $DF_2$ is the factor "beam width" (in one tenth of millimeter). From the procedure listed in Appendix F, we first generate data on quality attributes (listed in Appendix G) that is distorted slightly to keep the expected score be close to the true score. By using the data in Appendix G and data on design factors listed in the Appendix E, we develop fuzzy and non-fuzzy multivariate regression models respectively. For developing the fuzzy model, the fuzzy input parameters are the five linguistic scales defined as in Table 4.2. For developing the non-fuzzy model, we take the central values of the membership functions corresponding to five linguistic scales as input parameters: "Excellent" = 10, "Good" = 7.5, "Fair" = 5, "Poor" = 2.5, and "Terrible" = 0. In a sense, when using numerical points to match with the qualitative scales, fuzziness is not taken into consideration. Both fuzzy and non-fuzzy models being developed are in the form as

$$QA_1 = B_{01} + B_{11}\ DF_1 + B_{21}\ DF_2$$

$$QA_2 = B_{02} + B_{12}\ DF_1 + B_{22}\ DF_2$$

where variables $QA_1$, $QA_2$, $DF_1$, and $DF_2$ are defined previously. After implementing fuzzy and non-fuzzy multivariate regression analysis, we obtain the following results :

Model parameters associated with $QA_1$

| | True Model | Non-fuzzy Model | Fuzzy Model |
|---|---|---|---|
| $B_{01}$ | 10 | 9.27 | [ 0.99 8.56 16.00] |
| $B_{11}$ | 6 | 7.63 | [-19.14 6.91 33.22] |
| $B_{21}$ | -3 | -3.20 | [ -7.59 -2.79 1.95] |

Model parameters associated with $QA_2$

| | True Model | Non-fuzzy Model | Fuzzy Model |
|---|---|---|---|
| $B_{02}$ | -3 | -2.94 | [-11.06 -2.46 5.62] |
| $B_{12}$ | 8 | 7.96 | [-20.17 9.23 35.64] |
| $B_{22}$ | 1 | 0.99 | [ -4.41 0.33 6.72] |
| $\Lambda$ | | 0.00005 | [0.0005 0.0025 0.0081] . |

From the above result, we recognize that both fuzzy and non-fuzzy models are consistent with the true model. It indicates that when there is no or insignificant distortion in the data on consumer distribution, the multivariate regression analysis is robust with or without incorporating with fuzzy sets. In other words, when all the consumers have used the right scales for evaluation, which results the distribution that closely reflects the true scores of products, the obtained non-fuzzy regression model is as good as fuzzy one.

In the above example, since the central (or mode) value of each corresponding membership function is chosen to be a non-fuzzy input parameter, the parameter is in fact the special case of fuzzy input parameter. Therefore, obtained non-fuzzy model is actually a special outcome of fuzzy model. It is surmised that when the evaluation distribution of consumers are distorted to a more significant degree or the non-fuzzy input parameters deviate from the central values of membership functions, the resulting non-fuzzy model may digress from the assumed true model while the fuzzy model is still robust. In the following, two cases are presented to demonstrate the advantage of fuzzy multivariate regression analysis for model development.

Case 1. Suppose we distort the evaluation distribution of consumers to a higher degree and obtain data that is listed in Appendix H, then follow the procedure of multivariate regression analysis with and without incorporating with fuzzy sets to obtain the following results.

Model parameters associated with $QA_1$

| | True Model | Non-fuzzy Model | Fuzzy Model |
|---|---|---|---|
| $B_{01}$ | 10 | 8.47 | [ -1.22 8.60 18.21] |
| $B_{11}$ | 6 | 7.83 | [-28.83 5.05 35.13] |
| $B_{21}$ | -3 | -3.94 | [ -8.52 -3.11 2.94] |

Model parameters associated with $QA_2$

| | True Model | Non-fuzzy Model | Fuzzy Model |
|---|---|---|---|
| $B_{02}$ | -3 | -0.06 | [ -9.46 -0.34 8.20] |
| $B_{12}$ | 8 | -1.76 | [-27.90 2.86 30.09] |
| $B_{22}$ | 1 | 2.83 | [ -3.36 1.52 8.09] |
| $\Lambda$ | | 0.00005 | [0.0011 0.0041 0.0109] |

From the above results, we acknowledge that several parameters of non-fuzzy model deviate from the true model to a substantial degree. More seriously, the coefficient $B_{12}$ of non-fuzzy model has a sign that is opposite to the true model's, which shows that the non-fuzzy model fails to agree with the true model even $\Lambda$ indicates the model is fit. Comparatively, the fuzzy model is still robust since all the parameter values of true model fall within the range of fuzzy parameters.

Case 2. Suppose non-fuzzy input parameters deviate from the central values of membership functions of fuzzy input parameters. For example, the new input membership functions are defined as: "Excellent" = [9.5, 10, 10], "Good" = [7.5, 8.5, 9.5], "Fair" = [2.5, 5.0, 7.5], "Poor" = [0.5, 1.5, 2.5], and "Terrible" = [0, 0, 0.5]. Same as before, the non-fuzzy input parameters are: "Excellent" = 10, "Good" = 7.5, "Fair" = 5, "Poor" = 2.5, and "Terrible" = 0. A data set on quality attributes (listed in Appendix I) is generated according to the central values of the new membership functions. Following the procedure of multivariate regression analysis with and without incorporating with fuzzy sets, we obtain the following results.

Model parameters associated with $QA_1$

| | True Model | Non-fuzzy Model | Fuzzy Model |
|---|---|---|---|
| $B_{01}$ | 10 | 9.27 | [ 3.03 9.07 15.29] |
| $B_{11}$ | 6 | 4.29 | [-14.97 6.62 27.25] |
| $B_{21}$ | -3 | -2.40 | [ -7.00 -2.87 0.97] |

Model parameters associated with $QA_2$

| | True Model | Non-fuzzy Model | Fuzzy Model |
|---|---|---|---|
| $B_{02}$ | -3 | 0.10 | [ -8.13 -0.45 6.73] |
| $B_{12}$ | 8 | 0.44 | [-22.57 3.39 26.85] |
| $B_{22}$ | 1 | 1.57 | [ -2.90 1.20 6.87] |
| $\Lambda$ | | 0.00000002 | [0.0004 0.0016 0.0051] |

Again it is found that the obtained non-fuzzy model fails to agree with the true model because $B_{02}$ has an opposite sign while the fuzzy model is still robust. At this point, we conclude that fuzzy multivariate regression analysis is less sensitive to the distortion of evaluation distribution of consumers due to the capability of fuzzy sets in quantifying qualitative measurement, thus better coping with subjective uncertainty.

# CHAPTER VI

# CONCLUSIONS AND RECOMMENDATIONS

Product design is an iterative decision-making activity whereby scientific principles and structured technical information are used to specify features and design dimensions, as well as to select and interconnect materials and components. Quality acceptance in developed products, however, is in the eye of the product user and therefore, the primary goal of designing product is to meet the needs, wants, and capabilities of intended consumers. The design method introduced and practiced in this dissertation is focused on the consumer-oriented quality assurance. Several techniques that have been well developed in various fields are integrated into a process of analysis for product design. This integrated method may be employed in the design of products that have multiple quality attributes perceived by the consumer.

In this product design procedure, the identification of determinant quality attributes and product specification are performed sequentially. This method has the following functions:

1. It deals with product design and quality assurance as a total concept.
2. It uses up-front consumer analysis to specify design by providing a consumer-based perspective on quality.
3. It provides ways to quantify and measure consumer perceived quality.
4. It allows consumer perceived quality to be predictable if certain designs are specified.

## Summary

Manufacturing high-quality products requires understanding what the consumer wants and needs, which are exhibited by the product attributes. A good consumer-oriented analysis on the product's quality should provide assessment on attribute importance from the perspective of the consumer. Because a consumer's preference is revealed mainly by comparing products, and one's judgement is prone to subjectivity, the measurement of consumer perceived quality becomes critical. While consumer opinion is important, the evaluation on quality from the designer's perspective should not be overlooked due to designer's distinctive knowledge about the product and the limitations in production. The first focus of this research is on the development of a quantitative method which combines the evaluations from the consumer and the designer. It demonstrates that the integration of several analytical tools, i.e., fuzzy entropy method and eigenvector method, provides a methodical procedure to facilitate the function of ranking a product's quality attributes.

According to the product's determinant quality attributes, the designer's role is to make the product's specifications be formed in concrete. In other words, the product's dimensions and materials should be specified in a way to ensure or promote the quality of designed product. It is the second focus of this research to model the correlation between perceived product quality and the design specifications. Fuzzy multivariate regression analysis is applicable to develop this relationship model, which is then used to experiment postulated designs for predicting their quality levels. Accordingly, an optimal design can be specified by comparing predicted quality levels of different design alternatives.

In summary, the theme of this research is "quality begins with the customer". The problem solving scheme of the proposed method consists of (1) the identification of determinant product attributes, and (2) the specification of design to meet consumer preference. Neither the theme nor the scheme is really novel and many studies have addressed these issues. However, the intention of this research is to substantiate the abstract theme by utilizing consumer input explicitly in the design process.

## Contributions

The research developed a design methodology for products that possess multiple quality attributes perceived by the consumer. Inherent in this method is the modeling framework required to achieve a design that meets consumer preference. The following specific contributions are made by this research:

1. It developed a formal modeling and methodology framework that is applicable to designing consumer products.

2. It followed a coherent procedure for analysis such that it can be implemented to real problems of product design.

3. It considered and incorporated all the possible roles and relevant decision elements in decision-making scenarios.

4. The method's flexibility allows modifications for the confronted scenarios.

5. Based on reasonable assumptions, it utilizes suitable quantitative methods from various fields to solve the defined problems.

6. The application methods are intuitively attractive, and each has a strong theoretical support. They were also justified and validated for their application in solving the defined problems.

7. According to the design method, computer programs are developed.

8. It adopted a reliable technique, fuzzy sets methodology, for measuring consumer perceived quality. This application has an immense potential in both marketing research and engineering design.

## Recommendations

This research provides an analytical method for assisting product design from the consumer's perspective. As a complemental technique for design methodology and quality assurance, this method should be incorporated with other techniques. Also because of the method's unconventionality for product design, several issues need further study.

The integrity of input data for analysis is considered to be the most important issue for implementing the proposed method. Only when input data is credible, can the method give reliable results. Since the measurement on quality level of a product comes from consumer evaluation, organizing the focus group of representative consumers needs to be addressed.

In this research, it is assumed that the perceptions of designers are homogeneous. Therefore, it is suggested that the pair-wise comparison matrix for eigenvector method can be formed by a single designer. The assumption, however, may not hold for a real case. Therefore, eliciting and integrating the judgements from multiple designers becomes another important issue. Either quantitative techniques, such as decision making theory on group decision, or qualitative techniques, such as Delphi technique, may provide a good solution.

Throughout the developed method, fuzzy sets methodology is applied and membership functions are used to represent qualitative perceptions or linguistic scales. In fuzzy sets theory, there is no standard way for constructing membership functions. In this study, a formula, i.e., $\pi$ function, with assumed parametric values is used for the construction of membership functions. For general applications, however, actual construction of membership functions through an experiment or a survey may be preferred. In addition, the degree of fuzziness of a membership function actually depends on the value of range of variation. This value is so critical that it may affect the shape and range of output membership function(s). Sensitivity analysis should be conducted to understand how the range of variation affects the results and how it should be determined.

As mentioned in Chapter IV, a fuzzy multivariate regression model is used for predicting the quality level with postulated values of design factors. A regression model is repeatedly used for various design specifications until the predicted quality is acceptable. This approach is seen as an enumeration method which may not result in a global optimum. In this aspect, certain optimization techniques may be applicable to specify design factors. In the theory of fuzzy sets, fuzzy mathematical programming has been addressed and several algorithms have been proposed (66). Incorporation of the design method developed in this research with a formal optimization technique needs further study.

# APPENDICES

## Appendix A

## Procedure of Multivariate Regression Analysis

Defining **Y** to be the $n$ x $p$ data matrix of $n$ independent observations on $p$ responses, **X** to be the $n$ x $(q+1)$ design matrix of fixed known $q$ predictor variables, **B** to be the $(q+1)$ x $p$ matrix of parameters to be estimated, and **E** to be the matrix of random errors, the multivariate linear model for multivariae regression analysis is

$$\underset{(nxp)}{\mathbf{Y}} = \underset{(nx(q+1))}{\mathbf{X}} \; \underset{((q+1)xp)}{\mathbf{B}} + \underset{(nxp)}{\mathbf{E}}$$

with mean of **Y**, $E(\mathbf{Y}) = \mathbf{XB}$, and variance-covariance of **Y**, $V(\mathbf{Y}) = \mathbf{I} \otimes \Sigma$, where $\Sigma$ is variance-covariance matrix.

For any choice of parameters **B**, the matrix of errors is **Y** - **XB**. The error sum of squares and cross-products matrix is $(\mathbf{Y} - \mathbf{XB})^T(\mathbf{Y} - \mathbf{XB})$. With the given outcomes **Y** and the values of **X**, the matrix of parameters **B**, which minimize the trace (the sum of the diagonal elements) of the matrix $(\mathbf{Y} - \mathbf{XB})^T(\mathbf{Y} - \mathbf{XB})$, is acquired by estimation:

$$\underset{((q+1)xp)}{\mathbf{B}\wedge} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

with $E(\mathbf{B}\wedge) = \mathbf{B}$ and $V(\mathbf{B}\wedge) = \Sigma \otimes (\mathbf{X}^T\mathbf{X})^{-1}$.

Using the least squares estimates **B^**, we can form the matrices of

$$\text{Predicted values: } \mathbf{Y}\wedge = \mathbf{XB}\wedge = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

$$\text{Residuals: } \mathbf{E}\wedge = \mathbf{Y} - \mathbf{Y}\wedge = [\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T]\mathbf{Y}$$

The orthogonality conditions among the residuals, predicted values, and columns of **X**, which hold in classical linear regression, hold in multivariate multiple regression. They follow from $\mathbf{X}^T[\mathbf{I}-\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T] = \mathbf{X}^T - \mathbf{X}^T = \mathbf{0}$. Specifically,

$$\mathbf{X}^T\mathbf{E}\wedge = \mathbf{X}^T[\mathbf{I}-\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T]\mathbf{Y} = \mathbf{0}$$

so the residuals $\mathbf{E}_{(i)}\wedge$ are perpendicular to the columns of **X**. Also $\mathbf{Y}\wedge^T\mathbf{E}\wedge = \mathbf{B}\wedge^T\mathbf{X}^T[\mathbf{I}-\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T]\mathbf{Y} = \mathbf{0}$ confirming that the predicted values $\mathbf{Y}_{(i)}\wedge$ are perpendicular to all residual vectors $\mathbf{E}_{(k)}\wedge$. Because $\mathbf{Y} = \mathbf{Y}\wedge + \mathbf{E}\wedge$,

$$Y^TY = (Y\hat{} + E\hat{})^T(Y\hat{} + E\hat{}) = Y\hat{}^TY\hat{} + E\hat{}^TE\hat{} + 0 + 0^T, \text{ or}$$

$$Y^TY = Y\hat{}^TY\hat{} + E\hat{}^TE\hat{}$$

where

$Y^TY$ is the total sum of squares and cross-products,

$Y\hat{}^TY\hat{}$ the predicted sum of squares and cross-products,

$E\hat{}^TE\hat{}$ the error sum of squares and cross-products.

The predicted sum of squares and cross-products, and the residual sum of squares and cross-products can also be written respectively as

$$Y\hat{}^TY\hat{} = B\hat{}^TX^TXB\hat{}$$

$$E\hat{}^TE\hat{} = Y^TY - Y\hat{}^TY\hat{} = Y^TY - B\hat{}^TX^TXB\hat{}.$$

Suppose the joint test on the significance of parameter $H_o$: $B = 0$ is of interest, a multivariate analysis-of-variance (MANOVA) table as below is used for the hypothesis test.

| Source | df | SS | *E*(MS) |
|---|---|---|---|
| Total Regression | $q+1$ | $W_h = B\hat{}^TX^TXB\hat{}$ | $\Sigma + (B^TX^TXB)/(q+1)$ |
| Residual | $n-q-1$ | $W_e = Y^TY - B\hat{}^TX^TXB\hat{}$ | $\Sigma$ |
| Total | $n$ | $W_t = Y^TY$ | |

To test the hypothesis, several criteria are available. For this research, the most commonly used Wilks' Lambda criterion is used for its simplicity in calculation, and it has a detailed criterion table available to check with (58). The hypothesis $H_o$: $B = 0$ is rejected at the significance $\alpha$ if

$$\Lambda = \frac{|W_e|}{|W_e + W_h|} < \Lambda_{critical}(\alpha; p, q+1, n-q-1)$$

where $|W|$ is the determinant of $W$, and $\Lambda_{critical}(\alpha; p, q+1, n-q-1)$ is the Wilks' Lambda criterion with three parameters.

To test the hypothesis of using a subset of predictor variables, a procedure can be followed to test for the significance of the difference between two Lambdas. The Wilks' Lambda from the step with the complete set or larger number of variables ($\Lambda_2$) is divided by the Lambda from the step with the subset of fewer variables ($\Lambda_1$), producing a new value of $\Lambda_D$. In the formula form, $\Lambda_D = \Lambda_2 / \Lambda_1$.

This operation is to eliminate the effect of the predictor variables not included in the subset. Because the fewer predictor variables for calculating $\Lambda_1$ , the greater the value of $\Lambda_1$ will be; and the more predictor variables for calculating $\Lambda_2$, the smaller the value of $\Lambda_2$. Thus the value of $\Lambda_D$ will remain between 0 and 1. The significance of $\Lambda_D$ is evaluated against $\Lambda_{critical}$ ($\alpha$; $p, q_S+1, n-q_S-1$), where $q_S$ is the number of predictor variables of subset, and $q_S < q$.

In multivariate analysis, the test of no linear relationship between the two complete sets of variables is most often of concern, and its hypothesis test is $H_o$: $\Gamma = \mathbf{0}$. For this test, the same procedure of testing the significance of the difference between two Lambdas can be applied. In this case, the number of predictor variables to calculate $\Lambda_1$ is 0, accordingly the significance of $B_0$ will be excluded from testing linear relationship. Once $\Lambda_D$ is obtained, the linear relationship is tested by evaluating $\Lambda_D$ against $\Lambda_{critical}$ ($\alpha$; $p$, $q$, $n-q-1$).

Same as univariate regression, $R^2$ (coefficient of determination) can be used in multivariate regression as a descriptive measure to assess the goodness of fit of an assumed regression model. Its value is computed separately for each equation associated with one response variable to study the effectiveness of each relationship in accounting for observed variation. A low value of $R^2$ often means some important variables have been omitted, or assumed form of the regression relation is inadequate. The effect of each predictor variable on $R^2$ can be examined, and variables which contribute little to $R^2$ are eliminated.

# Appendix B

## Computer Programs

### B1. Program Description for the Developed Procedure

B1.1 Fortran program for ranking quality attributes by the procedure of entropy method.

A. *Description.*

This program, ENTROPY.FOR, is used to rank quality attributes based on consumer perspective. It utilizes Subroutines RANDOM and FUNC for de-fuzzying input membership functions (fuzzy numbers) into crisp numbers, and Subroutine STATIS for calculating statistical parameters, curve-fitting, and reconstr ucting fuzzy numbers. This process is based on the six-step JHE method, which is described in the text. The access to the source code of these subroutines is under the permission of Dr. C. H. Juang, primary author of the JHE method and source code.

B. *Limitations.*

The maximum number of linguistic scales is 5. The maximum number of quality attributes is 10, and the maximum number of alternatives is 30. The dimensions of variables can be changed according to need.

B1.2 Fortran program for ranking quality attributes by the procedure of eigenvector method.

A. *Description.*

This program, EIGEN.FOR, is used to rank quality attributes based on the designer perspective.

B. *Limitations.*

The maximum number of quality attributes is 10. The dimensions of variables can be changed according to need.

B1.3 Fortran program for ranking quality attributes by the procedure of ranking integration.

A. *Description.*

This program, RANK.FOR, is used to rank quality attributes by integrating consumers' and the designer's evaluation. It utilizes Subroutine RANDOMN to generate uniform random numbers, subroutine ROOT to back calculate the upper bound x if the integration is known as a random number, and subroutine STATIS to calculate statistical parameters and to reconstruct fuzzy numbers. This process is based on JHE method, described in the text. The access to the source code of these subroutines is under the permission of Dr. C. H. Juang, primary author of JHE method and source code.

B. *Limitations.*

The maximum number of quality attributes is 10. The dimensions of variables can be changed according to need. Iteration number of simulation may be added as an additional dimension for some declared variables, and the do-loop should be added to the program for the simulation.

B1.4 Fortran program for multivariate regression analysis.

A. *Description.*

This program, MVR.FOR, is used to model correlation between quality attributes and design factors. It utilizes Subroutines RANDOM and FUNC for converting input membership functions (fuzzy numbers) into crisp numbers, and Subroutines STATIS1 and STATIS2 for converting output crisp numbers into fuzzy numbers. This process is based on the JHE method described in the text. The access to the source code of these subroutines is under the permission of Dr. C. H. Juang, primary author of the JHE method and source code.

B. *Limitations.*

The maximum number for both quality attributes and design factors is 5. The maximum number of observations is 30. The dimensions of variables can be changed according to need.

## B2. Program Listing for the Developed Procedure

```
      program entropy

      integer noquat,noalt,xzero,h
      real x,sum(10,30),temp,perc(5,10,30),di(10),etemp,bigc
      real wi(10),eitemp,ditemp,sumt,esumt
      data (observ(i), i = 1,5)/ 'E','G','F','P','T'/

c noquat: no. of quality attributes (QA)
c noalt: no. of alternative products
c perct(i,j,k): the propotion of consumers use a linguistic term to evaluate a product
c             where i is no. of linguistic values,
c                   j is no. of quality attributes
c                   k is no. of products.
c observ(i): the initial of a linguistic value (eg. "E" for EXCELLENT)

      print,' Enter the file name for input data.'
      read 150, fname1
      open (unit=20,file=fname1)
      print,' Enter a file name for output.'
      read 150,fname2
150   format(a12)

      do 10 i = 1,noquat
            do 20 k = 1,noalt
                  read(20,100) perc(1,i,k),perc(2,i,k),perc(3,i,k),perc(4,i,k),perc(5,i,k)
100               format(f6.1,2x,f6.1,2x,f6.1,2x,f6.1,2x,f6.1)
20          continue
10    continue

      do 30 i = 1,noquat
            di(i) = 0.
            do 40 k = 1,noalt
                  sum(i,k) = 0.
40          continue
30    continue

      xzero = 1
      bigc = -1./log(real(noalt))

      do 50 i = 1,noquat
            ditemp=0.
            do 60 k = 1, noalt
                  sumt=0.
                  do 70 h = 1, 5
                        call random(rand,xzero)
                        call func(a(h),c(h),rand,x,observ(h))
                        sumt = sumt + perc(h,i,k)*x/100.
70                continue
                  sum(i,k) = sumt
                  ditemp = ditemp+sumt
60          continue
```

```
            di(i) = ditemp
            eitemp = 0.
            do 80 k = 1,noalt
                  temp = sum(i,k)/di(i)
                  sum(i,k) = temp
                  ei = temp*log(temp)
                  eitemp = eitemp + etemp
80          continue
            wi(i) = bigc*eitemp
50    continue

      esumt = 0.
      do 90 i = 1,noquat
            esumt = esumt+wi(l,i)
90    continue

      do 110 i = 1,noquat
            wi(i) = 1/(real(noquat)-esumt)*(1-wi(i))
            print 120,i,wi(i)
120         format(' Consumer Evaluated Weight w(',i2,')=',f7.5)
110   continue

      call statis(noalt,wi,mean,stad,noquat,wimin,xm,wimax,nocut,alpha,beta)
      open(unit=18,file=fname2)
      write(18,130) wimin(i),xm(i),wimax(i),alpha(i),beta(i)
130   format(2x,f7.5,2x,f7.5,2x,f7.5,2x,f7.5,2x,f7.5)

      end
```

```
        program eigen

        real lambda,sum
        character fname*12
        real a(10,10),x(10),d(10),z(10),w(10)
        data epsi/.000000000001/

        print,' Enter a File Name for Output'
        read 300,fname
300     format(a12)
        print,'How many attributes to weight?'
        read, n

        do 10 i = 1,n
                do 20 j = i,n
                        print,'What is the ratio of attribute',i
                        print,'                      of attribute',j
                        read(*,30) a(i,j)
20              continue
10      continue
30      format(f6.1)

        do 40 i = 2,n
                imini = i-1
                do 50 j = 1,imini
                        a(i,j) = 1/a(j,i)
50              continue
40      continue

        print, 'The matrix is as follows:'
        print 60 ((a(i,j), j = 1,n), i = 1,n)
60      format(' ',3f15.4)

        do 70 i = 1,n
                x(i) = 1.
70      continue
        it = 0
80      do 90 i = 1,n
                d(i) = 0.
                do 100 j = 1,n
                        d(i) = d(i)+a(i,j)*x(j)
100             continue
90      continue
        it = it+1
        do 110 i = 1,n
                z(i) = d(i)/d(1)
110     continue
        do 120 i = 1,n
                diff = x(i)-z(i)
                if(dabs(diff)-epsi*dabs(z(i))) 120,120,130
120     continue
```

```
      go to 150

130   do 140 i = 1,n
            x(i) = z(i)
140   continue
      if(it .ge. 100) go to 170
      go to 80
150   do 160 i = 1,n
      x(i) = z(i)
160   continue
170   lambda = d(1)
      print,'The largest eigenvalue is:'
      print 250, lambda
250   format(e14.3)

      sum = 0.0
      do 180 i = 1,n
            sum = sum+x(i)
180   continue
      do 190 i = 1,n
            w(i) = x(i)/sum
190   continue
      print 220
      open (unit=12,file=fname)
220   format('The relative weight of attributes are:'/)
      print 230 (w(i), i = 1,n)
      write(12,230) (w(i), i = 1,n)
230   format(2x,f7.5)

      close(12)

      end
```

```
      program rank

      real wcmin(10),wcmax(10),wd(10),rand,alpha(10),beta(10)
      real wi(10),xm(10),xc(10),denom,wic,mean(10),stad(10)
      integer noquat,i,l,xzero,nocut,noalt,noint
      character fname1*12,fname2*12,fname3*12

      print,' Enter File Name from Entropy Method'
      read 200, fname1
200   format(a12)

      print,' Enter File Name from Eigenvector Method'
      read 200, fname2
      print,' Enter No. of Iteration'
      read, nocut
      print,' Enter a File Name for Output File'
      read 200, fname3
      xzero=1
      open(unit=12,file=fname1)
      open(unit=14,file=fname2)

      do 10 i=1,noquat
            read(12,210) wcmin(i),xm(i),wcmax(i),alpha(i),beta(i)
210         format(2x,f7.5,2x,f7.5,2x,f7.5,2x,f7.5,2x,f7.5)
            read(14,220) wd(i)
220         format(2x,f7.5)
10    continue

      do 40 l=1,nocut
            print,'Iteration:',l
            denom=0.
            do 50 i=1,noquat
                  call randomn(xzero,rand)
                  call root(wcmin(i),wcmax(i),alpha(i),beta(i),rand,wic)
                  xc(i)=wic
                  denom=denom+wic*wd(i)
50          continue
      do 60 i=1,noquat
            wi(l,i)=xc(i)*wd(i)/denom
            print 230,i,wi(l,i)
230         format(' Wi(',i2,')=',f7.5)
60    continue
40    continue

      call statis(wi,wcmin,wcmax,mean,stad,alpha,beta,nocut,noquat,fname)

      end
```

```
      program  mtvreg

      real xtrasp(5,30),nom(10),ytemp(5,30,6),av(5),cv(5)
      integer noy,nox,noob,noitr,m,xo
      character flag*1,fname*12,observ(5),fname1*12
      real b(30,5),y(30,5),xinv(5,5),x(30,5),ytras(5,30),ytemp1,
     x xprd(5,5),btemp(30,5),det1,det2,lamda,yestras(5,30)
      data (observ(i), i = 1,5)/ 'E','G','F','P','T'/

      xo = 1

      print,'Input the number of response variables: QAs'
      read, noy
      print,'Input the number of predictor variables: DFs'
      read, nox

      nox = nox+1
      print,'Input the number of observations'
      read, noob
      print,'Input the file name for important QAs'
      read 400, fname
400   format(a12)

      do 500 i = 1, noy
            print,'Which QA is taken for analysis?'
            read, nom(i)
500   continue

      print,'Input the file name for design factors (DFs)'
      read 400, fname1
      open(unit=10,file=fname)
      open(unit=11,file=fname1)

      do 600 i = 1, noob
            read (11,*) (x(i,j), j = 2,nox)
600   continue

      print,'How many iterations for analysis'
      read, noitr
      do 700 i = 1,noy
            do 710 j = 1,nom(i)
                  do 720 k = 1,noob
                        read(10,410) (ytemp(i,k,l), l = 1,5)
410                     format(5(f6.1,2x))
720               continue
710         continue
            rewind(unit=10)
700   continue
```

```
        do 10 m = 1,noitr
                do 20 i = 1,noy
                        do 30 j = 1,noob
                        temp = 0.
                                do 40 k = 1,5
                                        call randomn(rand,xo)
                                        call func(av(k),cv(k),rand,xv,observ(k))
                                        temp = temp+xv*ytemp(i,j,k)/100.
40                              continue
                                y(j,i) = temp
30                      continue
20              continue
                do 50 i = 1, noob
                        x(i,1) = 1.
50              continue

                do 60 j = 1, noob
                        do 70 i = 1, nox
                                xtrasp(i,j) = x(j,i)
70                      continue
60              continue

                do 80 i = 1, nox
                        do 90 j = 1, nox
                                xprd(i,j) = 0.
                                do 100 k = 1, noob
                                        xprd(i,j) = xprd(i,j) +xtrasp(i,k)*x(k,j)
100                             continue
90                      continue
80              continue

                do 110 k = 1, nox
                        xprd(k,k) = -1./xprd(k,k)
                        do 120 i = 1, nox
                                if (i-k) 130, 120, 130
130                             xprd(i,k) = -xprd(i,k)*xprd(k,k)
120                     continue

                do 140 i = 1, nox
                        do 140 j = 1, nox
                                if ((i-k)*(j-k)) 150, 140, 150
150                             xprd(i,j) = xprd(i,j) - xprd(i,k)*xprd(k,j)
140             continue

                do 110 j = 1, nox
                        if (j-k) 160, 110, 160
160                     xprd(k,j) = -xprd(k,j)*xprd(k,k)
110             continue

                do 170 i = 1, nox
                        do 170 j = 1, nox
                                xinv(i,j) = -xprd(i,j)
170             continue
```

```
            do 180 i = 1, nox
                do 190 j = 1, noy
                    xprd(i,j) = 0.
                    do 200 k = 1, noob
                        xprd(i,j) = xprd(i,j) +xtrasp(i,k)*y(k,j)
200                 continue
190             continue
180         continue

            do 210 i = 1,nox
                do 220 j = 1,noy
                    b(m,i,j) = 0.
                    do 230 k = 1, nox
                        b(m,i,j) = b(m,i,j) + xinv(i,k)*xprd(k,j)
230                 continue
                    print,'Beta(',i,j,')=',b(m,i,j)
220             continue
210         continue

            do 240 k = 1, nox
                do 250 j = 1, noy
                    btemp(k,j) = b(m,k,j)
250             continue
240         continue

            do 260 i = 1,noob
                do 270 j = 1,noy
                    ytemp1 = 0.
                    do 280 k = 1, nox
                        ytemp1 = ytemp1 + x(i,k)*btemp(k,j)
280                 continue
                    b(m,i,j) = ytemp1
                    print,'yest(',i,j,')= ',b(m,i,j)
270             continue
260         continue

            do 290 i = 1,noob
                do 300 j = 1,noy
                    btemp(i,j) = b(m,i,j)
                    yestras(j,i) = b(m,i,j)
                    ytras(j,i) = y(i,j)
300             continue
290         continue
            do 310 i = 1,noy
                do 320 j = 1,noy
                    xprd(i,j)=0.
                    xinv(i,j)=0.
                    do 330 k = 1,noob
                        xprd(i,j)=xprd(i,j)+ytras(i,k)*y(k,j)
                        xinv(i,j) =xinv(i,j)+ytras(i,k)*y(k,j)-
     x                  yestras(i,k)*btemp(k,j)
330                 continue
320             continue
310         continue
```

```
          do 340 i = 1,noob
                 do 350 j = 1, noy
                        b(m,i,j) = y(i,j) - b(m,i,j)
                        print,'err(',i,j,')=',b(m,i,j)
350              continue
340       continue

          call determ(xinv,noy,det1)
          call determ(xprd,noy,det2)
          lamda(m) = det1/det2
          print,' lamda(',m,') = ',lamda(m)

10   continue

     call statis1(b,nox,noy,noitr,1)
     call statis1(b,noob,noy,noitr,2)
     call statis1(b,noob,noy,noitr,3)
     call statis2(lamda,noitr)

     close(10)
     close(11)

     end

     subroutine determ(a,num,deter)

     real deter,a(5,5)
     integer num,kp1
     data eps/0.000001/

     deter = 1.
     do 10 k = 1,num
            deter = deter*a(k,k)
            if(abs(a(k,k)).gt. eps) then
                   go to 20
            else
                   print,'Matrix may be singular'
                   go to 50
            endif
20          kp1 = k+1
            do 30 j = kp1,num
30                 a(k,j) = a(k,j)/a(k,k)
                   a(k,k) = 1.
            do 10 i = 1,num
                   if(i.eq.k .or.a(i,k).eq.0.) goto 10
            do 40  j = kp1,num
40          a(i,j) = a(i,j) - a(i,k)*a(k,j)
            a(i,k) = 0.
10   continue
     print,'Determinant of the matrix = ',deter

50   end
```

# Appendix C

## Sample Data File of Quality Attributes Evaluated by Consumers

### $QA_1$ (Maneuverability of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 32 | 4 | 30 | 13 | 22 |
| T2 | 4 | 4 | 25 | 55 | 12 |
| T3 | 3 | 34 | 6 | 13 | 44 |
| T4 | 20 | 40 | 28 | 5 | 8 |
| T5 | 18 | 14 | 28 | 22 | 17 |
| T6 | 21 | 29 | 20 | 17 | 12 |
| T7 | 22 | 36 | 12 | 9 | 21 |
| T8 | 8 | 7 | 4 | 45 | 35 |
| T9 | 11 | 15 | 18 | 7 | 49 |
| T10 | 9 | 22 | 37 | 18 | 14 |
| T11 | 15 | 21 | 24 | 18 | 22 |
| T12 | 0 | 13 | 24 | 26 | 38 |
| T13 | 33 | 5 | 25 | 31 | 6 |
| T14 | 27 | 29 | 19 | 8 | 17 |
| T15 | 19 | 12 | 31 | 29 | 9 |
| T16 | 27 | 28 | 37 | 4 | 4 |
| T17 | 30 | 23 | 30 | 15 | 3 |
| T18 | 10 | 24 | 18 | 42 | 6 |
| T19 | 25 | 4 | 5 | 36 | 30 |
| T20 | 5 | 61 | 28 | 4 | 3 |
| T21 | 20 | 21 | 21 | 14 | 25 |
| T22 | 36 | 2 | 24 | 15 | 24 |
| T23 | 0 | 2 | 5 | 5 | 88 |
| T24 | 16 | 27 | 26 | 8 | 23 |
| T25 | 38 | 9 | 41 | 0 | 12 |
| T26 | 19 | 17 | 23 | 6 | 35 |
| T27 | 21 | 10 | 52 | 9 | 8 |
| T28 | 10 | 24 | 18 | 19 | 30 |
| T29 | 6 | 25 | 18 | 39 | 12 |
| T30 | 19 | 12 | 6 | 18 | 45 |

**$QA_2$ (Power of tennis racquet)**

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 13 | 24 | 20 | 27 | 16 |
| T2 | 33 | 10 | 27 | 25 | 5 |
| T3 | 21 | 27 | 23 | 24 | 6 |
| T4 | 5 | 30 | 22 | 16 | 27 |
| T5 | 18 | 26 | 28 | 5 | 23 |
| T6 | 15 | 17 | 27 | 18 | 24 |
| T7 | 15 | 14 | 26 | 22 | 22 |
| T8 | 35 | 27 | 11 | 20 | 7 |
| T9 | 27 | 26 | 22 | 15 | 10 |
| T10 | 21 | 24 | 18 | 25 | 12 |
| T11 | 11 | 26 | 11 | 26 | 26 |
| T12 | 32 | 32 | 20 | 6 | 11 |
| T13 | 15 | 30 | 2 | 26 | 27 |
| T14 | 11 | 18 | 24 | 29 | 18 |
| T15 | 38 | 5 | 6 | 27 | 24 |
| T16 | 4 | 6 | 41 | 20 | 30 |
| T17 | 15 | 16 | 7 | 30 | 32 |
| T18 | 14 | 35 | 6 | 26 | 19 |
| T19 | 21 | 23 | 24 | 28 | 4 |
| T20 | 10 | 7 | 19 | 62 | 2 |
| T21 | 7 | 32 | 31 | 26 | 4 |
| T22 | 18 | 18 | 15 | 36 | 12 |
| T23 | 39 | 11 | 42 | 0 | 9 |
| T24 | 26 | 10 | 32 | 4 | 28 |
| T25 | 2 | 20 | 11 | 43 | 24 |
| T26 | 25 | 29 | 4 | 22 | 19 |
| T27 | 25 | 14 | 13 | 21 | 27 |
| T28 | 14 | 32 | 25 | 26 | 3 |
| T29 | 27 | 10 | 31 | 28 | 4 |
| T30 | 40 | 1 | 27 | 9 | 22 |

**QA$_3$ (Stiffness of tennis racquet)**

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 7 | 15 | 24 | 42 | 11 |
| T2 | 26 | 21 | 10 | 28 | 16 |
| T3 | 20 | 28 | 9 | 12 | 30 |
| T4 | 35 | 25 | 26 | 11 | 3 |
| T5 | 33 | 39 | 5 | 20 | 3 |
| T6 | 33 | 29 | 22 | 3 | 14 |
| T7 | 36 | 20 | 6 | 22 | 16 |
| T8 | 18 | 23 | 5 | 27 | 27 |
| T9 | 27 | 13 | 15 | 30 | 16 |
| T10 | 17 | 15 | 24 | 26 | 18 |
| T11 | 17 | 0 | 19 | 31 | 32 |
| T12 | 16 | 39 | 1 | 37 | 6 |
| T13 | 29 | 24 | 30 | 2 | 16 |
| T14 | 32 | 16 | 4 | 24 | 25 |
| T15 | 13 | 33 | 6 | 14 | 35 |
| T16 | 20 | 4 | 7 | 44 | 25 |
| T17 | 19 | 32 | 16 | 11 | 23 |
| T18 | 0 | 7 | 30 | 40 | 23 |
| T19 | 6 | 24 | 20 | 32 | 19 |
| T20 | 13 | 26 | 13 | 41 | 7 |
| T21 | 8 | 26 | 8 | 11 | 47 |
| T22 | 25 | 24 | 23 | 1 | 27 |
| T23 | 27 | 12 | 1 | 45 | 16 |
| T24 | 8 | 28 | 30 | 22 | 12 |
| T25 | 22 | 27 | 15 | 19 | 18 |
| T26 | 30 | 4 | 26 | 24 | 17 |
| T27 | 15 | 10 | 20 | 17 | 37 |
| T28 | 27 | 13 | 26 | 21 | 13 |
| T29 | 20 | 19 | 22 | 23 | 16 |
| T30 | 22 | 19 | 25 | 17 | 17 |

QA$_4$ (Shock and Vibration Damping of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 16 | 23 | 22 | 5 | 34 |
| T2 | 5 | 29 | 27 | 21 | 18 |
| T3 | 38 | 6 | 19 | 17 | 20 |
| T4 | 11 | 22 | 38 | 4 | 25 |
| T5 | 24 | 5 | 20 | 17 | 35 |
| T6 | 16 | 22 | 24 | 38 | 1 |
| T7 | 29 | 21 | 24 | 11 | 15 |
| T8 | 51 | 2 | 11 | 30 | 6 |
| T9 | 15 | 1 | 43 | 30 | 12 |
| T10 | 14 | 2 | 18 | 43 | 24 |
| T11 | 5 | 21 | 5 | 32 | 37 |
| T12 | 12 | 33 | 13 | 41 | 0 |
| T13 | 19 | 23 | 3 | 23 | 31 |
| T14 | 30 | 30 | 12 | 28 | 0 |
| T15 | 23 | 18 | 9 | 27 | 23 |
| T16 | 38 | 24 | 7 | 24 | 7 |
| T17 | 25 | 32 | 1 | 16 | 26 |
| T18 | 14 | 21 | 12 | 25 | 28 |
| T19 | 8 | 25 | 43 | 14 | 10 |
| T20 | 23 | 16 | 21 | 22 | 18 |
| T21 | 0 | 17 | 26 | 26 | 30 |
| T22 | 11 | 8 | 36 | 13 | 31 |
| T23 | 23 | 11 | 35 | 2 | 29 |
| T24 | 5 | 10 | 16 | 7 | 62 |
| T25 | 26 | 18 | 8 | 21 | 27 |
| T26 | 28 | 15 | 29 | 15 | 13 |
| T27 | 19 | 32 | 2 | 40 | 7 |
| T28 | 18 | 25 | 19 | 25 | 13 |
| T29 | 24 | 26 | 24 | 4 | 21 |
| T30 | 6 | 18 | 43 | 8 | 25 |

QA$_5$ (Ball Control of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 35 | 6 | 34 | 18 | 8 |
| T2 | 28 | 3 | 27 | 18 | 24 |
| T3 | 19 | 59 | 7 | 13 | 1 |
| T4 | 14 | 10 | 34 | 11 | 30 |
| T5 | 17 | 29 | 11 | 28 | 15 |
| T6 | 35 | 27 | 22 | 4 | 12 |
| T7 | 24 | 15 | 19 | 20 | 22 |
| T8 | 35 | 10 | 9 | 21 | 26 |
| T9 | 14 | 2 | 36 | 32 | 15 |
| T10 | 25 | 22 | 13 | 23 | 17 |
| T11 | 32 | 20 | 11 | 9 | 28 |
| T12 | 7 | 22 | 32 | 6 | 32 |
| T13 | 18 | 22 | 22 | 12 | 26 |
| T14 | 23 | 30 | 0 | 23 | 24 |
| T15 | 39 | 15 | 6 | 28 | 12 |
| T16 | 5 | 5 | 44 | 1 | 45 |
| T17 | 14 | 33 | 19 | 3 | 30 |
| T18 | 12 | 32 | 8 | 25 | 23 |
| T19 | 19 | 9 | 18 | 25 | 29 |
| T20 | 18 | 26 | 11 | 29 | 16 |
| T21 | 31 | 7 | 22 | 9 | 31 |
| T22 | 5 | 20 | 19 | 20 | 36 |
| T23 | 22 | 20 | 24 | 29 | 6 |
| T24 | 22 | 19 | 32 | 25 | 3 |
| T25 | 20 | 29 | 22 | 18 | 12 |
| T26 | 11 | 24 | 27 | 11 | 27 |
| T27 | 11 | 36 | 7 | 3 | 43 |
| T28 | 10 | 11 | 35 | 30 | 14 |
| T29 | 21 | 24 | 27 | 4 | 24 |
| T30 | 35 | 2 | 13 | 26 | 25 |

## Appendix D

### Sample Data File of Quality Attributes Evaluated by the Designer

| | $QA_1$ | $QA_2$ | $QA_3$ | $QA_4$ | $QA_5$ § |
|---|---|---|---|---|---|
| $QA_1$ | 1 | 1/3 | 1/2 | 2 | 4 |
| $QA_2$ | 3 | 1 | 3 | 5 | 7 |
| $QA_3$ | 2 | 1/3 | 1 | 3 | 4 |
| $QA_4$ | 1/2 | 1/5 | 1/3 | 1 | 2 |
| $QA_5$ | 1/4 | 1/7 | 1/4 | 1/2 | 1 |

§
$QA_1$: Maneuverability
$QA_2$: Power
$QA_3$: Stiffness
$QA_4$: Shock and vibration damping
$QA_5$: Ball control

# Appendix E

## Sample Data File of Design Factors

(Design Factors of tennis racquet)

| Racquet | $DF_1$ | $DF_2$ | $DF_3$ | $DF_4$ | $DF_5$ | $DF_6$ | $DF_7$ | $DF_8$ § |
|---|---|---|---|---|---|---|---|---|
| T1 | 105 | 24 | 16 | 19 | 1 | 0 | 0 | 304 |
| T2 | 118 | 31 | 16 | 18 | 0 | 1 | 0 | 288 |
| T3 | 114 | 28 | 16 | 20 | 1 | 0 | 0 | 320 |
| T4 | 95 | 22 | 18 | 19 | 1 | 0 | 0 | 342 |
| T5 | 112 | 27 | 18 | 19 | 0 | 1 | 0 | 342 |
| T6 | 100 | 23 | 18 | 20 | 0 | 0 | 1 | 360 |
| T7 | 99 | 23 | 16 | 19 | 1 | 0 | 0 | 304 |
| T8 | 122 | 32 | 16 | 19 | 0 | 1 | 0 | 304 |
| T9 | 119 | 31 | 18 | 20 | 0 | 1 | 0 | 360 |
| T10 | 113 | 27 | 16 | 20 | 1 | 0 | 0 | 320 |
| T11 | 93 | 22 | 16 | 19 | 0 | 1 | 0 | 304 |
| T12 | 125 | 33 | 16 | 19 | 1 | 0 | 0 | 304 |
| T13 | 98 | 22 | 16 | 19 | 1 | 0 | 0 | 304 |
| T14 | 97 | 22 | 16 | 18 | 0 | 1 | 0 | 288 |
| T15 | 110 | 26 | 18 | 21 | 1 | 0 | 0 | 378 |
| T16 | 89 | 20 | 17 | 21 | 0 | 0 | 1 | 357 |
| T17 | 90 | 21 | 18 | 20 | 0 | 1 | 0 | 360 |
| T18 | 109 | 25 | 16 | 17 | 0 | 1 | 0 | 272 |
| T19 | 117 | 30 | 17 | 19 | 1 | 0 | 0 | 323 |
| T20 | 92 | 21 | 18 | 19 | 0 | 0 | 1 | 342 |
| T21 | 112 | 27 | 16 | 19 | 1 | 0 | 0 | 304 |
| T22 | 107 | 24 | 16 | 18 | 0 | 0 | 1 | 288 |
| T23 | 128 | 35 | 16 | 19 | 1 | 0 | 0 | 304 |
| T24 | 111 | 26 | 18 | 20 | 0 | 1 | 0 | 360 |
| T25 | 89 | 21 | 18 | 20 | 0 | 1 | 0 | 360 |
| T26 | 113 | 27 | 16 | 18 | 1 | 0 | 0 | 288 |
| T27 | 98 | 23 | 16 | 19 | 0 | 1 | 0 | 304 |
| T28 | 116 | 30 | 16 | 18 | 1 | 0 | 0 | 288 |
| T29 | 116 | 30 | 16 | 19 | 0 | 1 | 0 | 304 |
| T30 | 115 | 29 | 14 | 18 | 0 | 0 | 1 | 252 |

§
$DF_1$: Hitting area
$DF_2$: Beam width
$DF_3$: No. of main strings
$DF_4$: No. of cross strings
$DF_5$: Material is graphite
$DF_6$: Material is fiberglass
$DF_7$: Material is ceramic
$DF_8$: main strings x cross strings

# Appendix F

## Procedure for Generating Distribution of Consumer Evaluation

This procedure is used to generate distributions of consumer evaluation. The distribution associated with each product alternative is assumed to be a triangular distribution. A random variable X has a triangular distribution if its probability density function is given by

$$f(x) = [2(x - a)]/[(m - a)(b - a)], \quad a \le x \le m$$
$$= [2(b - x)]/[(b - m)(b - a)], \quad m < x \le b$$

where $a \le m \le b$; $a$ is the minimum, $b$ is the maximum, and $m$ is the mode of distribution. The expected value $E(X)$ is equal to $1/3(a + m + b)$. The cumulative distribution function, which measures the probability that the random variable X assumes a value less than or equal to x, for the triangular distribution is given by

$$F(x) = (x - a)^2/[(m - a)(b - a)], \quad a < x \le m$$
$$= 1 - (b - x)^2/[(b - m)(b - a)], \quad m < x \le b$$

Step 1. Generate true scores ($S_i$, i = 1 to 30) of $QA_1$ and $QA_2$ for thirty alternative products by plugging the values of $DF_1$ and $DF_2$ (listed in Appendix E) into the model that has been assumed to be true.

Step 2. The true score ($S_i$) of each alternative on a quality attribute is treated as the expected value $E(X)$, therefore the mode value, $m$, is equal to $3S_i - a - b$, where the values of $a$ and $b$ are predetermined by setting them to be close to the minimum (0) and maximum (10) of the range of universe of discourse for membership functions. Accordingly, the triangular distribution is fit with determined $a$, $b$, and $m$.

Step 3. Assign a value that is close to the central value of a scale's membership function to random variable X to determine F(x), which is the probability that the scale assumes.

Step 4. Repaet Step 3 until every scale, from "excellent" to "terrible" assumes a probability (percentage) that consumers use a certain scale to evaluate an alternative. Therefore, according to a quality attribute, the evaluation distribution of consumers is determined for an alternative.

Step 5. Repeat Step 2, 3, and 4 until the distribution associated with every alternative is determined.

Note: The degree of distortion of evaluation distribution depends on the difference between the expected value obtained from the distribution and the true score.

# Appendix G

## Sample Data File of Quality Attributes: Slightly Distorted from True Distribution

$QA_1$ (Maneuverability of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 34 | 33 | 22 | 10 | 1 |
| T2 | 5 | 38 | 36 | 18 | 2 |
| T3 | 15 | 41 | 27 | 13 | 1 |
| T4 | 36 | 31 | 20 | 10 | 1 |
| T5 | 21 | 39 | 26 | 13 | 2 |
| T6 | 35 | 32 | 21 | 11 | 1 |
| T7 | 35 | 32 | 21 | 11 | 1 |
| T8 | 4 | 35 | 39 | 19 | 2 |
| T9 | 5 | 39 | 36 | 18 | 2 |
| T10 | 22 | 38 | 25 | 13 | 2 |
| T11 | 35 | 32 | 21 | 11 | 1 |
| T12 | 4 | 31 | 42 | 21 | 3 |
| T13 | 39 | 30 | 20 | 10 | 1 |
| T14 | 38 | 30 | 20 | 10 | 1 |
| T15 | 26 | 36 | 24 | 12 | 2 |
| T16 | 42 | 29 | 19 | 10 | 1 |
| T17 | 38 | 30 | 20 | 10 | 1 |
| T18 | 31 | 34 | 23 | 11 | 1 |
| T19 | 7 | 42 | 33 | 16 | 2 |
| T20 | 39 | 30 | 20 | 10 | 1 |
| T21 | 21 | 39 | 26 | 13 | 2 |
| T22 | 35 | 32 | 21 | 11 | 1 |
| T23 | 3 | 23 | 43 | 28 | 4 |
| T24 | 27 | 36 | 24 | 12 | 2 |
| T25 | 37 | 31 | 20 | 10 | 1 |
| T26 | 22 | 38 | 25 | 13 | 2 |
| T27 | 34 | 33 | 22 | 11 | 1 |
| T28 | 6 | 42 | 33 | 17 | 2 |
| T29 | 6 | 42 | 33 | 17 | 2 |
| T30 | 9 | 44 | 30 | 15 | 2 |

QA$_2$ (Power of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 4 | 29 | 43 | 22 | 3 |
| T2 | 23 | 38 | 25 | 13 | 2 |
| T3 | 10 | 44 | 30 | 15 | 2 |
| T4 | 2 | 19 | 38 | 36 | 5 |
| T5 | 7 | 43 | 32 | 16 | 2 |
| T6 | 3 | 23 | 43 | 27 | 3 |
| T7 | 3 | 22 | 43 | 28 | 4 |
| T8 | 30 | 34 | 23 | 11 | 1 |
| T9 | 25 | 37 | 25 | 12 | 2 |
| T10 | 8 | 43 | 32 | 16 | 2 |
| T11 | 2 | 18 | 36 | 38 | 5 |
| T12 | 35 | 32 | 21 | 11 | 1 |
| T13 | 3 | 21 | 41 | 32 | 4 |
| T14 | 3 | 20 | 40 | 33 | 4 |
| T15 | 5 | 40 | 35 | 18 | 2 |
| T16 | 2 | 15 | 31 | 44 | 9 |
| T17 | 2 | 16 | 33 | 43 | 7 |
| T18 | 5 | 37 | 37 | 19 | 2 |
| T19 | 19 | 40 | 26 | 13 | 2 |
| T20 | 2 | 17 | 34 | 41 | 6 |
| T21 | 7 | 43 | 32 | 16 | 2 |
| T22 | 4 | 31 | 41 | 21 | 3 |
| T23 | 40 | 29 | 20 | 10 | 1 |
| T24 | 6 | 41 | 34 | 17 | 2 |
| T25 | 2 | 16 | 32 | 43 | 7 |
| T26 | 8 | 43 | 32 | 16 | 2 |
| T27 | 3 | 22 | 42 | 30 | 4 |
| T28 | 18 | 40 | 27 | 13 | 2 |
| T29 | 18 | 40 | 27 | 13 | 2 |
| T30 | 14 | 42 | 28 | 14 | 2 |

# Appendix H

## Sample Data File of Quality Attributes: Highly Distorted from True Distribution

$QA_1$ (Maneuverability of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 10 | 20 | 30 | 27 | 13 |
| T2 | 7 | 13 | 20 | 27 | 33 |
| T3 | 6 | 14 | 21 | 26 | 33 |
| T4 | 11 | 22 | 33 | 22 | 11 |
| T5 | 8 | 15 | 23 | 30 | 25 |
| T6 | 11 | 21 | 32 | 25 | 12 |
| T7 | 10 | 21 | 31 | 25 | 13 |
| T8 | 7 | 13 | 20 | 27 | 33 |
| T9 | 5 | 15 | 20 | 29 | 31 |
| T10 | 8 | 14 | 23 | 31 | 24 |
| T11 | 11 | 22 | 33 | 23 | 11 |
| T12 | 7 | 13 | 20 | 27 | 33 |
| T13 | 12 | 24 | 32 | 22 | 11 |
| T14 | 11 | 23 | 33 | 21 | 12 |
| T15 | 8 | 16 | 24 | 32 | 19 |
| T16 | 15 | 29 | 28 | 19 | 9 |
| T17 | 12 | 25 | 32 | 21 | 11 |
| T18 | 9 | 18 | 27 | 31 | 15 |
| T19 | 7 | 13 | 20 | 27 | 33 |
| T20 | 13 | 26 | 31 | 21 | 10 |
| T21 | 8 | 15 | 23 | 30 | 25 |
| T22 | 10 | 20 | 30 | 26 | 13 |
| T23 | 7 | 13 | 20 | 27 | 33 |
| T24 | 8 | 16 | 25 | 33 | 18 |
| T25 | 12 | 24 | 32 | 21 | 11 |
| T26 | 8 | 15 | 23 | 31 | 24 |
| T27 | 10 | 21 | 31 | 26 | 13 |
| T28 | 7 | 13 | 20 | 27 | 33 |
| T29 | 5 | 16 | 22 | 25 | 33 |
| T30 | 6 | 14 | 22 | 27 | 31 |

**$QA_2$ (Power of tennis racquet)**

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 16 | 31 | 27 | 18 | 9 |
| T2 | 33 | 27 | 20 | 13 | 7 |
| T3 | 31 | 27 | 25 | 11 | 6 |
| T4 | 11 | 22 | 32 | 23 | 12 |
| T5 | 27 | 29 | 22 | 15 | 7 |
| T6 | 13 | 26 | 31 | 20 | 10 |
| T7 | 11 | 28 | 30 | 23 | 8 |
| T8 | 33 | 27 | 20 | 13 | 7 |
| T9 | 30 | 29 | 21 | 15 | 5 |
| T10 | 28 | 29 | 22 | 14 | 7 |
| T11 | 10 | 21 | 31 | 25 | 13 |
| T12 | 32 | 28 | 22 | 11 | 7 |
| T13 | 12 | 23 | 33 | 22 | 10 |
| T14 | 11 | 22 | 34 | 21 | 12 |
| T15 | 22 | 31 | 23 | 16 | 8 |
| T16 | 9 | 18 | 27 | 31 | 15 |
| T17 | 10 | 19 | 29 | 29 | 14 |
| T18 | 20 | 32 | 24 | 16 | 8 |
| T19 | 33 | 27 | 20 | 13 | 7 |
| T20 | 10 | 20 | 30 | 27 | 13 |
| T21 | 27 | 29 | 22 | 15 | 7 |
| T22 | 17 | 33 | 25 | 17 | 8 |
| T23 | 33 | 27 | 20 | 13 | 7 |
| T24 | 24 | 31 | 23 | 15 | 8 |
| T25 | 9 | 19 | 28 | 29 | 15 |
| T26 | 28 | 29 | 22 | 14 | 7 |
| T27 | 12 | 24 | 32 | 21 | 11 |
| T28 | 33 | 27 | 20 | 13 | 7 |
| T29 | 31 | 29 | 20 | 14 | 8 |
| T30 | 33 | 26 | 21 | 12 | 8 |

## Appendix I

## Sample Data File of Quality Attributes: According to New Membership Functions

### $QA_1$ (Maneuverability of tennis racquet)

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 22 | 30 | 43 | 5 | 0 |
| T2 | 1 | 19 | 71 | 9 | 0 |
| T3 | 3 | 35 | 55 | 7 | 0 |
| T4 | 25 | 28 | 41 | 5 | 0 |
| T5 | 7 | 35 | 52 | 6 | 0 |
| T6 | 24 | 29 | 42 | 5 | 0 |
| T7 | 23 | 29 | 43 | 5 | 0 |
| T8 | 1 | 17 | 73 | 10 | 0 |
| T9 | 1 | 20 | 70 | 9 | 0 |
| T10 | 8 | 35 | 51 | 6 | 0 |
| T11 | 24 | 29 | 42 | 5 | 0 |
| T12 | 1 | 15 | 74 | 11 | 0 |
| T13 | 28 | 27 | 40 | 5 | 0 |
| T14 | 27 | 28 | 41 | 5 | 0 |
| T15 | 12 | 33 | 48 | 6 | 0 |
| T16 | 31 | 26 | 38 | 5 | 0 |
| T17 | 27 | 28 | 41 | 5 | 0 |
| T18 | 19 | 30 | 45 | 6 | 0 |
| T19 | 1 | 25 | 65 | 8 | 0 |
| T20 | 28 | 27 | 40 | 5 | 0 |
| T21 | 7 | 35 | 52 | 6 | 0 |
| T22 | 23 | 29 | 42 | 5 | 0 |
| T23 | 1 | 11 | 75 | 14 | 0 |
| T24 | 14 | 32 | 48 | 6 | 0 |
| T25 | 26 | 28 | 41 | 5 | 0 |
| T26 | 8 | 35 | 51 | 6 | 0 |
| T27 | 22 | 30 | 43 | 5 | 0 |
| T28 | 1 | 24 | 67 | 8 | 0 |
| T29 | 1 | 24 | 67 | 8 | 0 |
| T30 | 1 | 31 | 61 | 8 | 0 |

**$QA_2$ (Power of tennis racquet)**

**datum is in percentage**

| Racquet | Excellent | Good | Fair | Poor | Terrible |
|---|---|---|---|---|---|
| T1 | 1 | 14 | 75 | 11 | 0 |
| T2 | 10 | 34 | 50 | 6 | 0 |
| T3 | 2 | 31 | 60 | 7 | 0 |
| T4 | 0 | 9 | 72 | 18 | 0 |
| T5 | 1 | 26 | 64 | 8 | 0 |
| T6 | 1 | 11 | 75 | 14 | 0 |
| T7 | 0 | 11 | 75 | 14 | 0 |
| T8 | 18 | 31 | 46 | 6 | 0 |
| T9 | 11 | 34 | 49 | 6 | 0 |
| T10 | 1 | 28 | 63 | 8 | 0 |
| T11 | 0 | 9 | 71 | 20 | 0 |
| T12 | 23 | 29 | 43 | 5 | 0 |
| T13 | 0 | 9 | 72 | 19 | 0 |
| T14 | 0 | 10 | 73 | 17 | 0 |
| T15 | 1 | 21 | 70 | 9 | 0 |
| T16 | 0 | 7 | 61 | 31 | 0 |
| T17 | 0 | 8 | 65 | 27 | 0 |
| T18 | 1 | 18 | 72 | 9 | 0 |
| T19 | 5 | 36 | 53 | 7 | 0 |
| T20 | 0 | 8 | 68 | 24 | 0 |
| T21 | 1 | 26 | 64 | 8 | 0 |
| T22 | 1 | 15 | 74 | 10 | 0 |
| T23 | 30 | 27 | 39 | 5 | 0 |
| T24 | 1 | 22 | 68 | 9 | 0 |
| T25 | 0 | 8 | 64 | 29 | 0 |
| T26 | 1 | 28 | 63 | 8 | 0 |
| T27 | 0 | 10 | 74 | 15 | 0 |
| T28 | 4 | 36 | 54 | 7 | 0 |
| T29 | 4 | 36 | 54 | 7 | 0 |
| T30 | 2 | 34 | 56 | 7 | 0 |

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